Information Asymmetries
and
an Endogenous Productivity Reversion Mechanism

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February 2009

Abstract:
Several studies among recent empirical work have suggested that the systematic behavior of lending standards over the business cycle, with laxer standards applied during expansions and tighter standards applied during recessions, may be responsible for driving economic fluctuations. We build a dynamic screening model with informational asymmetries in credit markets that rationalizes these findings and generates endogenous fluctuations of total output and productivity via the lending standards channel. When the capital stock is high, which evolves endogenously, liquidity is high for all types of producers, allowing even the unproductive type to meet the early payments on the loan, and thus making signals about entrepreneurs' quality, inferred from such payments, less informative. The early payment required to accomplish screening out the unproductive types thus rises. Because the early payment hurts productive entrepreneurs by lowering their investments, competition among lenders results in the emergence of pooling contracts with no early payment requirement. Low productivity entrepreneurs enter production along with productive types, the composition effect setting off a recession. The opposite happens for low enough values of capital.

JEL Codes: E32, E44, D24
Keywords: Endogenous cycles, aggregate productivity reversion, lending standards, credit market frictions, information asymmetries

Acknowledgements:
We are grateful to Gary Biglaiser, Larry Jones, Peter Norman, Adriano Rampini, Balazs Szentes as well as seminar participants at the University of Chicago, Duke, Clemson, UNC, Brown and Universidad de Chile for their comments.

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1. Introduction

Several studies among recent empirical work have suggested that the systematic behavior of lending standards[1] over the business cycle may be responsible for the reversion of trends in aggregate productivity. Laxer standards during economic booms allow for unproductive firms to enter the market, reducing aggregate productivity and leading to defaults in subsequent periods. On the contrary, tight credit standards during economic downturns tend to exclude bad projects, thus sowing the seeds of an economic recovery.

Asea and Blomberg (1998) use a panel data set of two million commercial and industrial loans to find that laxer lending standards occur during expansions and tighter standards occur during recessions, and that such behavior considerably influences the dynamics of aggregate fluctuations. Lown and Morgan (2006) use a survey of loan officers to document similar systematic behavior of lending standards. Moreover, they point out that loan standards are more important than loan rates in explaining variation of business loans and output in the time series. Consistent with these empirical findings, the aggregate data reveals that delinquency rates and loan charge-off rates,[2] which we use as a measure of default rates, lag behind the business cycle, rising after expansions and falling after recessions (Figure 1).

Such behavior of lending standards is bound to have an impact on the economy, due to a widespread practice to finance production through banks. In fact, non-corporate business, representing roughly 1/3 of the total U.S. business net worth, relies entirely on bank loans. Although corporate business has considerably reduced its dependence on bank financing, it still holds over 25% of its debt in bank loans and mortgages and, as recently as in the mid-1970s, held around 45% of its debt in these instruments.[3] Because of the large extent of bank financing, the seeming imperfections arising from credit standards that are either too lax or too tight, have generated much concern among the policy makers. Alan Greenspan, speaking at the Chicago Bank Structure Conference in 2001, alarmingly stated that "the worst loans are made at the top of the business cycle...[and at the bottom] the problem is not making bad loans, it is not making any loans, whether good or bad...".

In this paper, we build a dynamic model, with fully rational agents, capable of generating the systematic behavior of lending standards. Lax standards at the top of the cycle allow entry of unproductive entrepreneurs, inducing a downturn in the economy. This situation in turn produces conditions conducive to the emergence of more stringent lending standards. We are therefore able to rationalize the systematic behavior of lending standards and produce a channel through which it can contribute to the turning of expansions into recessions and vice versa, as found empirically by Asea and Blomberg (1998). Our model does not rely on non-fully rational expectations on the part of loan officers, as in Berger and Udell

[1] Lending standards are contract terms other than the interest rate that are used to screen borrowers, for example, loan size or credit line limit, time to loan maturity or first payment towards the loan balance, or borrowers' balance sheet variables.

[2] Charge-offs, which are the value of loans removed from the books and charged against loss reserves, are measured net of recoveries as a percentage of average loans and annualized. Delinquent loans are those past due thirty days or more and still accruing interest as well as those in nonaccrual status. They are measured as a percentage of end-of-period loans. Source: The Board of Governors, downloaded from http://www.federalreserve.gov/releases/chargeoff/default.htm. GDP and GDP deflator data are taken from the Bureau of Economic Analysis.

[3] The source for the aggregate balanced sheet data for corporate and non-corporate business is The Board of Governors, downloaded from http://www.federalreserve.gov/releases/z1/Current/data.htm.

(2004), or explanations based on an exogenous reversal of aggregate productivity, which cannot capture the causal effect from lending standards to aggregate productivity.

In order to convey some intuition regarding the channel through which the (endogenously determined) state of the economy influences the lending standards, we first provide a few details about the model. There are two privately known types of entrepreneurs that differ in their productivity. Entrepreneurs produce capital in two stages, for both of which they must seek external financing. Banks screen entrepreneurs by requiring an early payment towards the loan balance upon completion of the first stage of production. We interpret the early loan payment requirement as a stylized version of the lending standards.[4] This payment can be always set high enough so that the unproductive entrepreneurs cannot afford it. Making this payment, however, lowers entrepreneurs' reinvestment into the second stage of production. Entrepreneurs who obtain financing for both stages of production can default after the second stage of production, in which case they abscond with a fraction of their output. Along the equilibrium paths analyzed in this paper, both entrepreneurs seek financing whenever the size of the early payment allows it, but only good entrepreneurs repay in full, while bad entrepreneurs default.

In our model of adverse selection with a competitive banking sector, we draw on Hellwig's (1987) result that equilibrium contracts are selected as the best contracts for the good type subject to the lenders' zero profit condition. Changes of lending standards arise from an endogenously evolving cost of effective screening. Two forces deliver pooling contracts at the top of the cycle (for high capital levels) and separating contracts, with unproductive entrepreneurs unfinanced, at the bottom. First, at peaks, when all entrepreneurs enjoy higher liquidity, the early payment required to accomplish screening out the unproductive types is high, and hence good entrepreneurs must be hurt more to ensure viable separation. Good entrepreneurs are then more willing to accept a pooling contract with no screening, which allows for higher reinvestment, even though asks for a higher than risk-free interest rate as a cross-subsidy. Second, pooling contracts that allows bad entrepreneurs to enter are less costly at peaks, as the bank recovers more money after default, and hence a lower interest rate is needed to ensure bank participation.

The basic fact captured by our model is that a given signal about an entrepreneur's productivity (a particular amount met as early payment) is informative at the trough, but not informative at the peak. Consequently, screening out the bad projects becomes more costly at the peak and separating contracts are beaten by pooling contracts with no screening at all. Low productivity entrepreneurs enter production along with productive types, the composition effect setting off a recession and a rise in default rates. This situation, in turn, eventually generates conditions – a low enough level of capital and liquidity – conducive to the emergence of separating contracts, thereby increasing productivity in the capital good sector, and leading to an economic recovery.

Hence, our model of information asymmetries in credit markets can give rise to endogenous fluctuations of capital, total output and productivity via the lending standards channel, rationalizing the empirical findings discussed above. Note that our model does not require an exogenous technological shock relating project failures to changes in the macroeconomic conditions in order to trigger a recession and high default rates after a period of high output. It thus can capture the *causal* effect of lending standards

[4] In the empirical work of Asea and Blomberg (2004), how early loan payments are due is included in the definition of a lending standard.

on aggregate activity, as emphasized by Asea and Blomberg (1998). In fact, it is not the case that in our model ex-ante good projects are financed and become ex-post bad investments due to a shock (which was presumably deemed a low probability event) resulting in bankruptcy. Instead, *ex-ante bad projects* are financed because screening is too costly. Financing of ex-ante bad projects is probably best exemplified with the dot-com mania of the late nineties, during which projects with no discernible sources of revenue were heavily financed. These were not good projects going bust due to an exogenous shock, if anything, the late nineties saw a strong technology improvement in the computer and internet industries. A more likely explanation is that ex-ante bad projects were financed because screening was particularly expensive.[5]

Thus, we develop a new insight into why lending standards vary with the state of the economy and may contribute to turning expansions into recessions and vice versa. The rest of the paper is organized as follows. Section II overviews related literature. In Section III, we introduce the general model, derive static equilibrium contracts for given prices and define the dynamic equilibrium. In Section IV, we study a fully dynamic economy with externalities in the production sector (which simplifies the analysis by making the price of capital constant) and show the existence of equilibrium paths along which the model economy exhibits cyclical behavior. The general model with no externalities is analyzed in Section V, where we find cyclical behavior and also the possibility of indeterminacy of equilibrium on some range of state variables (due to the existence of more than one self-fulfilling belief). Finally, in Section VI, we endogenize the amount of funds available to finance entrepreneurs' projects, and find the possibility of much richer dynamics and predictions that are qualitatively consistent with much of the empirical evidence: procyclicality of net worth, cash flows, investment and loanable funds, lower reliance on bank financing at the top of the cycle, default rates lagging after the business cycle, and finally, the possibility of slow recoveries and abrupt and severe recessions.[6] We conclude in Section VII.

2. Related Literature

A number of theoretical models illustrate potentially important interactions between informational frictions in credit markets[7] and economic fluctuations. For the purpose of our discussion, we focus on two strands of related work that pursue the idea that credit markets influence the course of the business cycle. One strand argues that credit market imperfections amplify exogenous shocks and make them more persistent. The other strand argues that credit market imperfections are responsible for a reversion in output.

[5] The recent subprime mortgage crisis also exemplifies poor screening standards (zero money down, low credit scores) arising in equilibrium.

[6] Although quantitative analysis is outside of the scope of this paper, one quick check of our model's potential quantitative relevance is to see how much of aggregate productivity variation over time transpires through the extensive margin (new firm entry/exit). In fact, Lee and Mukoyama (2008) document that in the manufacturing sector the extensive margin is slightly more important than the intensive margin for job creation/destruction over the cycle. Job creation by startups in booms is greater than net job creation of preexisting firms. Likewise, in downturns, job destruction by exiting firms is greater than net job destruction by continuing firms.

[7] The early works on informational frictions in credit markets are Stiglitz and Weiss (1981), Bester (1985) and De Meza and Webb (1987).

A well-known example of an amplification mechanism is Bernanke and Gertler (1989),[8] where the borrowers' balance sheets amplify exogenous external shocks in a model of costly state verification. Economic upturns improve borrowers' net worth, which lowers agency costs of financing investment, increases investment and hence amplifies the upturn, while the opposite happens in the presence of a downturn. Another example is Kiyotaki and Moore (1997), which assumes that loan payments cannot be enforced, and hence only collateralized debt arises in equilibrium. A temporary shock that reduces a credit constrained firm's net worth reduces this firm's ability to obtain new loans and therefore its investment, thus propagating the effect of a temporary shock. In Rampini (2004), entrepreneurial activity which consists in the undertaking of risky, but in expected terms productive, projects increases at peaks. This is due to agents' higher willingness to take risks when the economic situation is booming, and the smaller need for bearing such a risk in those times, due to slacker incentive constraints. Other studies focusing on the amplification of exogenous shocks arising from informational frictions include Williamson (1987), Greenwald and Stiglitz (1993), Bernanke, Gertler and Gilchrist (1999).

The second strand of literature, under which our paper falls, include Suarez and Sussman (1997), Reichlin and Siconolfi (2003) and Azariadis and Smith (1998). Suarez and Sussman (1997) generate a reversion mechanism that works through the effect of equilibrium prices on liquidity constraints. The model is a dynamic extension of the Stiglitz-Weiss (1982) model of lending under moral hazard to an overlapping generations model with three generations. During booms, old entrepreneurs sell high quantities and, as a consequence, prices are low implying that young entrepreneurs must finance a higher fraction of output externally. Because external financing generates excessive risk-taking, booms are followed by high project failure rates. Although it delivers an endogenous reversion mechanism, the main channel through which this mechanism works – higher reliance on external financing at peaks – appears to be at odds with the data (see Levy and Henessey 2007).

Reichlin and Siconolfi (2003) generalize the Rothschild and Stiglitz (1976) adverse selection problem by including moral hazard. Both safe and risky projects can be implemented. Entrepreneurs differ only in their return to implementing the risky project, with lower skilled entrepreneurs facing a higher fixed cost of implementing it. The safe project requires a zero fixed cost and hence yields higher expected returns. They embed this mechanism in an overlapping generations model where the opportunity cost of lending evolves endogenously. They show that endogenous cycles may arise: when loanable funds are high, equilibrium contracts are such that a large fraction of entrepreneurs engages into risky production, high setup costs decrease output and wages sending the economy into a recession. As in our model, the reversion mechanism is driven by the composition effect: risky projects (that are worse than safe projects in expected terms) are chosen at the top of the cycle causing a recession. In fact, the opposite assumption, i.e. that risky projects are better in expected terms, is made in Rampini (2004), and the informational frictions deliver an amplification mechanism. Another related paper that generates a reversion mechanism is Martin (2006). The mechanism in that model delivers investment level driven cycles, while we focus on the composition of producers' as a driving force behind aggregate productivity. In Azariadis and Smith (1998), the presence of adverse selection leads to the indeterminacy of equilibrium. Savers moving their funds in and out of the banking system, as a result of their anticipation of the real interest rate, may

[8] Carlstrom and Fuerst (1997) perform the quantitative analysis of the mechanism in Bernanke and Gertler (1989).

lead to endogenous cycles.

Similar in spirit to our work is Dell'Ariccia and Marquez (2006), which considers the effect of an exogenous shock to screening costs. This shock, which they associate with financial liberalization (capital inflow) leads to a deterioration of lending standards, deterioration of loan portfolios for the bank and possibly a banking crisis, just as observed in many of the emerging economies. Their idea that large screening costs lead to the selection of pooling contracts is similar to ours, the main difference being that in our model the screening costs arise endogenously.

3. The Model

3.1. Environment

Consider a model economy where time is discrete and indexed by $t = 0, 1, 2....$ It is populated with overlapping generations of entrepreneurs who live for two periods and there exist two types of goods: consumption and capital. When young, entrepreneurs are endowed with 1 unit of time and an ability to implement projects that produce capital. We assume entrepreneurs do not suffer disutility from labor and enjoy utility from consuming both when young and old, according to homothetic preferences represented by $u(c^y, c^o)$. A savings technology is available to them at the risk free rate R_f.

The consumption good is produced by overlapping generations of competitively behaved firms that live for two periods. A representative firm born in period $t-1$ purchases capital from the young entrepreneurs in period $t-1$ at price ρ_t and uses it in production along with labor in period t, according to technology $F(K_t, L_t, t) = A_t K_t^{\beta} L_t^{1-\beta}$. Capital fully depreciates upon use. We assume that firms are born with nothing but can borrow at the risk free rate R_f in order to make capital purchases.[9, 10] Since both the decision about capital K_t and labor demanded L_t must be simultaneously made in period $t-1$ (when capital is purchased), firms must then form beliefs about the next period wage w_t. These beliefs must be consistent with the actual wage in the next period, which must in turn clear the labor market given the capital K_t bought by the consumption good sector in the previous period.

Capital goods are produced by entrepreneurs. Each generation consists of measure μ of type G and measure $1-\mu$ of type B. Types differ in their productivity and are private information. Each entrepreneur can implement a project within a single period, but in two stages. A fixed cost in the amount of M units of consumption goods is required at each stage of production. A project implemented by an entrepreneur of type i yields a fixed amount f_i of capital goods at the end of the first stage (we assume that investing more than M in the first stage yields no additional return) and $g_i + \frac{g_i}{M}s$ units of capital good at the end of the second stage, where s represents investment of funds beyond the fixed cost amount into the second stage of the project. We assume that type G is more productive at each stage.

Assumption 1 *$f_G > f_B$ and $g_G > g_B$.*

[9]Taking price ρ_t and beliefs w_t as given, a firm born in period $t-1$ solves $\max\limits_{K_t, L_t} A_t K_t^{\beta} L_t^{1-\beta} - R_f \rho_t K_t - w_t L_t$.

[10]We also assume that the firm born in period -1 is endowed with K_0 and a debt in the amount of its value $R_f F_1(K_0, 1, 0) K_0$.

There is a competitive banking sector that loans investment funds to the young entrepreneurs. Each period, banks are endowed with $2M\mu$ loanable units of the consumption good, exactly the fixed cost amount of implementing projects of all type G entrepreneurs. This particular amount is assumed for analytical simplicity.[11] We relax this assumption and endogenize the supply of funds in Section VI. A risk-free savings technology is available to the bank at rate R_f.

We consider contracts signed in the beginning of the entrepreneurs' young period. If an entrepreneur enters into a contract (δ, R), he receives an amount M in the beginning of the first stage and another amount M in the beginning of the second stage, conditional on meeting a partial payment δ towards the loan balance. At the end of the period, the remaining loan repayment is $2MR - \delta$. Whenever financing is obtained, the entrepreneur must implement the project, which is justified by the availability of a monitoring technology. We allow for default upon completion of the second stage, in which case a fraction α of wealth is kept by the entrepreneur, while a fraction $1-\alpha$ is seized by the bank. We interpret α as representing the state of available bankruptcy institutions. We assume that default can take place only at the end of the second stage. This captures the idea that it is more difficult to hide income during initial stages of production and is made only for analytical simplicity.[12]

Alternatively, we could assume that banks offer a credit line to entrepreneurs and contracts specify the credit limit and the interest rate on the borrowed funds. In the appendix, we prove that identical equilibrium outcomes are achieved under the assumption of contracts in the form of a credit line with credit limit $2M - \delta$ and interest rate $\frac{2MR-\delta}{2M-\delta}$.

It is instructive to examine the cash flows for an entrepreneur who enters into a contract (δ_t, R_t) at time t, reinvests everything into the second stage of production, and does not default. At time t, a young entrepreneur obtains M units of the consumption good from the bank in the beginning of the first stage of production and invests it into his project. At the end of the first stage he receives $\rho_{t+1} f_i$ in payment for his capital and w_t as labor income from supplying 1 unit of time to the the consumption good sector. He pays δ_t towards the loan balance upon completion of the first stage of production. He receives M units of the consumption good from the bank and invests into the second stage, along with his own income net of the loan payment, $w_t + \rho_{t+1} f_i - \delta_t$, which transforms into capital at the rate of $\frac{g_i}{M}$. At the end of the period, the entrepreneur sells his capital $g_i + (w_t + \rho_{t+1} f_i - \delta_t)\frac{g_i}{M}$ at price ρ_{t+1} and pays the remaining loan balance $2MR_t - \delta_t$ to the bank. The net worth at the end of the period, $\rho_{t+1}\left[g_i + \left(w_t + \rho_{t+1} f_i - \delta_t\right)\frac{g_i}{M}\right] - (2MR_t - \delta_t)$, is then allocated between consumption when young in t and consumption when old in $t+1$ according to u and R_f.

Two stages of production and the assumption that the consumption good sector delivers the payment for capital goods produced in the first stage before the second stage of production starts are necessary to allow for the possibility of screening. In our case, the screening tool is the requirement of a partial payment towards the loan balance before production is completed. Thus, we capture the idea that the lenders always have the ability to screen out bad entrepreneurs, because type G can always afford a higher

[11] With both types applying for loans, some crowding out of type G will take place as long as the amount of funds is less than $2M$, which is needed for the results derived under the present setup. However, our results also extend to a more general setup with any amount of loanable funds as long as financing of type B entrepreneurs replaces investment in some safe capital production technology with returns higher than those generated by B.

[12] This essentially means that in case of default at the end of the first stage, the bank seizes the entire wealth of the defaulting entrepreneur.

payment than type B. However, with liquidity (as determined by w_t or ρ_{t+1}) evolving endogenously through time, the cost of effective screening will also evolve endogenously through time, determining whether or not the screening tool is used in equilibrium at a particular point in time.

The previous discussion points out that our theory allocates an important role to the presence of liquidity positively correlated with the state of the economy. Although we chose to model this liquidity as arising due from wage income, a number of other modeling choices would suffice (e.g. income from land holdings).

3.2. Entrepreneurs' Behavior

For a given contract (δ_t, R_t) and prices w_t and ρ_{t+1}, a time t young entrepreneur of type i, who reinvests all the proceedings from the first stage (net of the early payment δ_t), chooses among the options summarized below. Since we focus on static choices, we drop the time subscripts for the rest of the section.

(O1) Do not enter into the contract. End of period net worth is w.

(O2) Enter into the contract, meet the payment δ, default. End of period net worth is $\alpha\rho\left[g_i + (w + \rho f_i - \delta)\frac{g_i}{M}\right]$.

(O3) Enter into the contract, meet the payment δ, pay in full. End of period net worth is $\rho\left[g_i + (w + \rho f_i - \delta)\frac{g_i}{M}\right] - (2MR - \delta)$.

To determine the best option, it suffices to compare the end of period net worth associated with each option. Note that since default is not an option before the second stage, entrepreneur i will always choose O1 if $\delta < w + \rho f_i$, i.e., if he cannot afford the early payment.

We denote the maximum early payment affordable by type B entrepreneurs by

$$\tilde{\delta}(w, \rho) := w + \rho f_B. \tag{1}$$

Note that $\tilde{\delta}(w, \rho)$ increases in both w and ρ, which raise labor and capital income respectively.

Both adverse selection (due to the existence of private information about types) and moral hazard (due to the possibility of not repaying the loan) arise in our model of competitive lending. We are, however, interested in equilibrium paths along which (i) type B entrepreneurs seek financing whenever the early payment is affordable ($\delta < \tilde{\delta}(w, \rho)$) and in case of obtaining it, default on the loan; (ii) type G entrepreneurs seek financing whenever the early payment is affordable ($\delta < w + \rho f_G$) and repay in full. Focusing on a particular behavior of entrepreneurs, i.e. a particular resolution of the moral hazard, allows us to use the results from Hellwig (1987) which apply to competitive markets with adverse selection. These results will enable us to determine equilibrium contracts for a given state of the economy.

In short, we will seek restrictions on the parameter space and the initial conditions that induce behavior (i) and (ii) along any equilibrium path. Clearly, the trade-offs that entrepreneurs face depend on the endogenously determined prices, which in turn depend on the state of the economy. In Assumption 2, we state sufficient conditions on prices w and ρ that guarantee existence of a non-empty set of contract menus that are incentive compatible, induce entrepreneurs to behave according to (i) and (ii) and yield nonnegative profits (see Lemma 1). When we analyze the dynamic behavior of the economy, we will derive parametric conditions and restrictions on the initial conditions such that Lemma 1 applies to the entire equilibrium path.

Assumption 2 *Suppose the following conditions hold for prices w, ρ:*
a). $\frac{\rho g_B}{M} > 1$.
b). $w + \rho f_B + (1-\alpha)\rho g_B \leq 2MR_f$.
c). If

$$\mu((1-\alpha)\frac{\rho g_G}{M} - 1) + (1-\mu)((1-\alpha)\frac{\rho g_B}{M} - 1) \leq 0, \tag{2}$$

then

$$2MR_f \leq (1-\alpha)\rho[\mu[g_G + (w+\rho f_G)\frac{g_G}{M}] + (1-\mu)[g_B + (w+\rho f_B)\frac{g_B}{M}]. \tag{3}$$

If (2) *fails, then*

$$2MR_f \leq \mu\left[(1-\alpha)\rho[g_G + \rho(f_G - f_B)\frac{g_G}{M}] + \rho f_B + w\right] + (1-\mu)\left[\rho f_B + w + (1-\alpha)\rho g_B\right]. \tag{4}$$

d). $\alpha\rho g_B \geq w$.
e). $1 > (1-\alpha)\frac{\rho g_B}{M}$.
f). $(1-\alpha)\rho g_B + w + \rho f_B < 2MR_f$.

Lemma 1 *Suppose that Assumptions 1 and 2 hold for prices (w, ρ). Define F as the set of menus $((R_B, \delta_B), (R_G, \delta_G))$ that induce self-selection, generate nonnegative profits, and such that $R_i \geq R_f$. The following hold for all menus in F:*

- *Both types, if financed, reinvest all the cash flows from the first stage (after the early payment δ) into second stage production.*

- *If $\delta_B < \tilde{\delta}(w, \rho)$ (i.e., the contract is affordable to type B), type B chooses to enter and default.*

Moreover, the set F is nonempty and for any $\delta_B \leq \tilde{\delta}(w, \rho)$, there exists an R_B and a contract (δ_G, R_G) such that the menu $((\delta_B, R_B), (\delta_G, R_G))$ is in F. In particular,

- *For any $\delta_B < \tilde{\delta}(w, \rho)$ the pooling menu $((\delta_B, \hat{R}), (\delta_B, \hat{R}))$ is in F for some uniquely determined $\hat{R} > R_f$ that generates zero profits, and it induces type G to enter and repay. Moreover, any menu $((\delta_B, R_B), (\delta_G, R_G)) \in F$ has $R_G \geq \hat{R}$, and it generates strictly positive profits if $R_G > \hat{R}$.*

- *For $\delta_B = \tilde{\delta}(w, \rho)$, the separating menu $((\tilde{\delta}(w, \rho), R_f), (\tilde{\delta}(w, \rho), R_f))$ is in F, it generates zero profits and induces type G to enter and repay.*

Proof. See the appendix. ■

The previous lemma asserts that in the set of feasible contract menus, type B enters and defaults. It also shows that it is always possible to find feasible contracts, either by excluding type B (by setting a high enough δ_B) or inducing type G to accept a contract under which he repays and cross-subsidizes type B through a pooling menu.

The lemma also states that any menu in F that involves an affordable contract for type B must involve a cross subsidy from type G ($R_G > R_f$). The pooling menu $\left((\delta_B, \hat{R}), (\delta_B, \hat{R})\right)$ guarantees exactly zero profits, but any menu in F with $R_G > \hat{R}$ generates strictly positive profits.

In what follows, we will show that in equilibrium, banks will offer either pooling contracts or contracts where type B is excluded. This result will greatly simplify the analysis.

3.3. Equilibrium contracts

Next we characterize equilibrium contracts that emerge for given prices w_t and ρ_{t+1}. As is known from Rothschild and Stiglitz (1976), equilibrium does not always exist in a context of competitive markets with adverse selection. The brief intuition underlying this result is as follows. If separating contracts are preferred by the good type, equilibrium exists. If, however, pooling contracts are preferred by the good type and therefore Pareto superior, competition imposes a zero-profit condition which induces cross-subsidization, and this implies that a bank offering such a contract is exposed to cream-skimming from a competitor targeting the good clients. Hellwig (1987) resolved this dilemma by incorporating a third stage into the game, so that after banks have offered contracts and agents have applied, banks have the possibility to reject their applicants, based on all the contracts offered by competing firms. Using the concept of sequential equilibrium, Hellwig proved that the pooling equilibrium that maximizes the utility of the good type subject to the zero-profit condition can be sustained.

When considering contracts in set F defined in Lemma 1, we are focusing on a particular resolution of the moral hazard. Therefore, Hellwig's results that apply to competitive markets with adverse selection apply to our setting, which allows us to follow the standard practice of finding equilibrium contracts $((\delta_G, R_G), (\delta_B, R_B))$, whether pooling or sorting, as maximizers of the utility of type G subject to the zero-profit condition.

The next lemma states that financing of both types cannot occur under different contracts. Intuitively, if both types receive financing through different contracts, then each one of these contracts must yield zero profit to be robust to cream-skimming. However, we already saw that type B never repays. Hence, the only way for type B to be financed is through a pooling contract.

Lemma 2 *If both types receive financing, then* $(\delta_B, R_B) = (\delta_G, R_G)$.

Proof. See the appendix. ■

We can thus restrict our attention to menus in F that specify identical contracts (δ, R) for both types, whether or not they exclude type B.

If a bank chooses to exclude type B entrepreneurs, by Lemma 1 it can do so by offering a single contract $(\tilde{\delta}(w, \rho), R_f)$ which induces type G to repay and yields zero profits.

Moreover, by Lemma 1, type B enters and defaults for any $\delta < \tilde{\delta}(w, \rho)$, but type G can be induced to cross-subsidize type B with a pooling contract that yields zero profits. Under this contract, both types enter, type G repays in full and type B defaults. Because types are unobservable and only $2M\mu$ is loaned out, a measure μ^2 of entrepreneurs of type G and a measure $(1-\mu)\mu$ of type B are financed. The funds obtained as loan repayment consist of full repayment from measure μ^2 of type G entrepreneurs and δ_B collected as early payment together with fraction $1-\alpha$ of output seized from measure $(1-\mu)\mu$ of type

B. Hence, the interest rate in this contract is a solution to

$$\mu^2 2MR + (1-\mu)\mu \left[\delta + (1-\alpha)\rho\left(g_B + (w + \rho f_B - \delta)\frac{g_B}{M}\right)\right] = \mu 2MR_f, \tag{5}$$

where the right hand side represents the opportunity cost of loaning $2M\mu$.

The zero profit condition associated with menus in F specifying a single contract such that $\delta \leq \tilde{\delta}(w,\rho)$ can be summarized by

$$R(w,\rho,\delta) = \begin{cases} \frac{R_f}{\mu} - \frac{(1-\mu)}{2M\mu}\left[\delta + (1-\alpha)\rho\left(g_B + (w+\rho f_B - \delta)\frac{g_B}{M}\right)\right], & \text{if } \delta < \tilde{\delta}(w,\rho) \\ R_f, & \text{if } \delta = \tilde{\delta}(w,\rho) \end{cases}, \tag{6}$$

where we solved (5) for R. Note that $R(w,\rho,\delta)$ is greater than R_f by Lemma 1 and decreases in both ρ and w in the interval $[0, \tilde{\delta}(w,\rho))$. Intuitively, as income increases, the amount recovered from type B increases, and hence a lower interest rate is needed to ensure zero profit for the lender. Moreover, $R(w,\rho,\delta)$ decreases in δ: condition (e) of Assumption 2 implies that an increase in δ (which lowers reinvestment for type B but decreases the amount he can abscond with) actually raises the total collection from type B and thus requires a lower cross-subsidy from type G under a pooling contract.

Under any contract, for given prices (w,ρ), the net worth obtained by type G is given by $\rho(g_G + (w + \rho f_G - \delta)\frac{g_G}{M}) - (2MR - \delta)$. Therefore, the equilibrium contract offered by banks is a solution to

$$\begin{aligned} \max_{(\delta,R)} \quad & \rho\left(g_G + (w+\rho f_G - \delta)\frac{g_G}{M}\right) - (2MR - \delta) \\ \text{s.t} \quad & \\ R \; &= \; R(w,\rho,\delta), \end{aligned}$$

where $R(w,\rho,\delta)$ is given by equation (6).

This problem, which yields the equilibrium contracts, is summarized by the feasible set (zero-profit condition) and type G indifference curves, depicted in Figure 2. To find the solution to this problem, we note that indifference curves describing his trade-off between R and δ, given current prices, have the slope

$$ICS := -\frac{\rho\frac{g_G}{M} - 1}{2M} < 0, \tag{7}$$

which is negative by condition (a) of Assumption 2. Moreover, indifference curves become steeper as ρ rises. This last fact is intuitive since, as capital sells for a higher price, the cost of foregone investment associated with δ rises, and type G requires a greater reduction in R to compensate for a given rise in δ. The solution depends on the relative size of ICS and the slope of the line connecting the following two points on the lender's zero-profit curve: the point associated with $\delta = 0$ (a pooling contract) and a point associated with $\tilde{\delta}(w,\rho)$ (a separating contract). We call this slope IPS, the first two letters referring to the iso-profit,

$$IPS := -\frac{R(w,\rho,0) - R_f}{\tilde{\delta}(w,\rho)} < 0. \tag{8}$$

It is easy to see from Figure 2 that the solution to the (linear) banking problem is that a pooling contract $(0, R(w,\rho,0))$ is offered if $ICS < IPS$, and a separating contract $(\tilde{\delta}(w,\rho), R_f)$ is offered otherwise.

Note that all of type G entrepreneurs (measure μ) and none of type B entrepreneurs are financed under the separating contract. Hence, total capital production amounts to the production of type G entrepreneurs:

$$k_s(\rho) = \mu\left(f_G + g_G + (\rho f_G + w - \tilde{\delta}(w,\rho))\frac{g_G}{M}\right) = \mu\left(f_G + g_G + (\rho(f_G - f_B))\frac{g_G}{M}\right). \tag{9}$$

Under the pooling contract, a measure μ^2 of type G entrepreneurs and a measure $\mu(1-\mu)$ of type B entrepreneurs obtain financing and enter production. Since the production of type i under the contract that involves no early payment is given by $f_i + g_i + (\rho f_i + w)\frac{g_i}{M}$, the total capital production is given by

$$k_p(w,\rho) = \mu^2\left(f_G + g_G + (\rho f_G + w)\frac{g_G}{M}\right) + \mu(1-\mu)\left(f_B + g_B + (\rho f_B + w)\frac{g_B}{M}\right). \tag{10}$$

The following proposition summarizes these results.

Proposition 1 *If* $\frac{1-\rho\frac{g_G}{M}}{2M} \geq -\frac{R(w,\rho,0)-R_f}{\tilde{\delta}(w,\rho)}$*, then a separating contract* $(\tilde{\delta}(w,\rho), R_f)$ *is offered, and total capital production is given by (9). Otherwise, a pooling contract* $(0, R(w,\rho,0))$ *is offered, and total capital production is given by (10).*

Proof. *Follows from the above discussion.* ■

3.4. Dynamic Equilibrium

Up to this point, we derived equilibrium contracts and capital production for given prices. As usual prices are determined endogenously, but in this case the consumption good firms producing in period $t+1$ must also form beliefs in period t about wages w_{t+1}, which are not announced in period t but influence the decision about the demand for capital k_{t+1}. These beliefs must be consistent with the actual wages, which in turn must clear the labor market in period $t+1$, given the total capital purchased by the consumption good sector k_{t+1}.

An expectation about future wages can then be seen as an expectation about the future level of capital k_{t+1}, which must be consistent with the actual decisions of entrepreneurs and firms in period t. This is the convention we use throughout the paper.

Definition 1 *A dynamic equilibrium for given* k_0 *is given by sequences of prices* $\{w_t^*, \rho_t^*\}_{t=0}^{\infty}$*, capital levels* $\{k_{t+1}^*\}_{t=0}^{\infty}$*, beliefs* $\{k_t'^*\}_{t=0}^{\infty}$ *and contracts* $\{R_t^*, \delta_t^*\}_{t=0}^{\infty}$ *such that the following holds:*

- *Expectations are consistent with equilibrium outcomes (rational):* $k_t'^* = k_{t+1}^*$,
- *Prices are given by* $w_t^* = A_t(1-\beta)k_t^{*\beta}$ *and* $\rho_{t+1}^* = A_{t+1}\beta\left(k_t'^*\right)^{\beta-1}/R_f$*, and satisfy Assumption 2.*
- *Contracts and capital production for given prices are determined according to Proposition 1, i.e.,*

$$(\delta_t^*, R_t^*) = \begin{cases} \left(\tilde{\delta}\left(w_t^*, \rho_{t+1}^*\right), R_f\right) & \text{if } \frac{1-\rho_{t+1}^*\frac{g_G}{M}}{2M} \geq -\frac{R\left(w_t^*,\rho_{t+1}^*,0\right)-R_f}{\tilde{\delta}\left(w_t^*,\rho_{t+1}^*\right)} \\ \left(0, R\left(w_t^*, \rho_{t+1}^*, 0\right)\right) & \text{otherwise} \end{cases},$$

where $\tilde{\delta}(w,\rho)$ *is defined in* (1) *and* $R(w_t^*, \rho_t^*, 0)$ *is defined in* (6)*;*

$$k_{t+1}^* = \begin{cases} k_s\left(\rho_{t+1}^*\right) & \textit{if } \frac{1-\rho_{t+1}^* \frac{g_G}{M}}{2M} \geq -\frac{R\left(w_t^*, \rho_{t+1}^*, 0\right) - R_f}{\tilde{\delta}\left(w_t^*, \rho_{t+1}^*\right)} \\ k_p\left(w_t^*, \rho_{t+1}^*\right) & \textit{otherwise} \end{cases},$$

where $k_s(\rho)$ *and* $k_p(w,\rho)$ *are given by* (9) *and* (10)*.*

4. Simplified Model

We now turn to study the existence of equilibrium in a simple case, where we assume the presence of a productivity externality due to the size of the economy. This makes the price of capital constant, while the wage w_t varies linearly with the capital level k_t. The reversion mechanism, which is the focus of this paper, is easy to observe in this context. In this case, liquidity changes arise only due to changes in labor income (and not changes in revenues from capital sales), and therefore expectations about future capital level do not play a role. Higher labor income raises the level of the early payment needed for effective screening. In addition, with higher labor income, default by type B entrepreneurs is less costly for the banks, implying that a lower interest rate is charged in case of pooling. These two forces combined make the pooling contract emerge for a high enough level of capital, the composition of producers subsequently dampening the future capital level. Similar intuition explains why a separating contract appears for a low enough level of capital.

4.1. Prices and Equilibrium Contracts

We now consider a positive externality on the production of consumption goods, by assuming $A_t = K_t^{\gamma}$, $\gamma + \beta = 1$. We then have

$$R_f \rho_t = \beta \text{ and } w_t = (1-\beta)\, k_t. \tag{11}$$

Since the rental price of capital is constant, k_t is the only state variable and expectations do not play any role. We drop the time subscript for this subsection. To keep the notation simple, from now on we also set $R_f = 1$.

Drawing on the earlier derivation of equilibrium contracts for given prices, we first obtain equilibrium contracts as a function of the state variable k. The minimum early payment that accomplishes separation becomes $\tilde{\delta}(k) = (1-\beta)\, k + \beta f_B$ and the maximization problem that determines the equilibrium contracts simplifies to

$$\begin{aligned} \max_{(\delta,R)} \quad & \rho\left(g_G + ((1-\beta)k + \beta f_G - \delta)\frac{g_G}{M}\right) - (2MR - \delta) \\ & \text{s.t.} \\ & R = R(k,\delta) = \begin{cases} \frac{R_f}{\mu} - \frac{(1-\mu)}{2M\mu}\left[\delta + (1-\alpha)\beta\left(g_B + ((1-\beta)k + \beta f_B - \delta)\frac{g_B}{M}\right)\right], & \text{if } \delta < \tilde{\delta}(k) \\ R_f, & \text{if } \delta = \tilde{\delta}(k) \end{cases}. \end{aligned}$$

As explained in the previous section, the solution to this problem is given by comparing the slope of the indifference curves (ICS) and the slope that represents the trade-off between R and δ in order to guarantee bank's participation (IPS). Expressions (7) and (8) now become

$$ICS = -\frac{\beta\frac{g_G}{M} - 1}{2M} \text{ and } IPS = -\frac{R(k,0) - R_f}{\tilde{\delta}\,(k)}.$$

Proposition 1, which characterized contracts offered in equilibrium, can be now expressed in terms of k. If $\frac{1-\beta\frac{g_G}{M}}{2M} \geq -\frac{R_k(0)-R_f}{\tilde{\delta}(k)}$, then a separating contract $(\tilde{\delta}\,(k)\,, R_f)$ is offered. Otherwise, a pooling contract $(0, R(k,0))$ is offered.

Note that ICS in independent of k, while IPS increases in k for two reasons. First, higher labor income raises $\tilde{\delta}\,(k)$, making it more costly to screen out type B entrepreneurs. Second, $R(k,0)$ decreases with higher income, because a higher collection from defaulting entrepreneurs allows for a lower interest rate to guarantee zero profits for the banks. Both effects make the separating contract (with a positive early payment $\tilde{\delta}\,(k)$ but a low interest rate R_f) less attractive for type G entrepreneurs when k is high. First, the higher early payment $\tilde{\delta}\,(k)$ needed to accomplish separation is costly for type G as it lowers his rate of reinvestment, making the separating contract less attractive. Second, a lower cross-subsidy in the pooling contract due to the lower interest rate $R_k\,(0)$ makes the pooling contract less attractive.

Note that there is a threshold $\bar{k}$, which divides the state space into regions of pooling and separating contracts.

Lemma 3 *Define* $\bar{k} = \left(\frac{(1-\mu)\left(2MR_f - g_B(1-\alpha)\beta\right)}{\mu(\beta\frac{g_G}{M}-1)+\beta\frac{g_B}{M}(1-\mu)(1-\alpha)} - \beta f_B\right)/\left(1-\beta\right)$. *For* $k > \bar{k}$, *a pooling contract is selected, while for* $k \leq \bar{k}$, *a separating contract is selected.*

Proof. Follows immediately from $\bar{k}$ being the unique solution to $\frac{\beta\frac{g_G}{M}-1}{2M} = \frac{R(k,0)-R_f}{\tilde{\delta}(k)}$. ■

Figure 3 illustrates the equilibrium contract determination for two arbitrary states of the economy, k_L and k_H such that $k_L < \bar{k} < k_H$.

4.2. Dynamic Equilibrium

We now focus on the existence of an equilibrium which exhibits cycles, which is the focus of the paper. In short, we seek the parametric restrictions and bounds on k_0 that give rise to cyclical equilibrium paths along which prices satisfy Assumption 2.

Total production, characterized in Proposition 1 for given prices, can also be determined as a function of k alone:

$$\begin{aligned} k_s &= \mu\left(f_G + g_G + \beta\left(f_G - f_B\right)\frac{g_G}{M}\right), \\ k_p\left(k\right) &= \mu^2\left(f_G + g_G + \left(\left(1-\beta\right)k + \beta f_G\right)\frac{g_G}{M}\right) + \left(1-\mu\right)\mu\left(f_G + g_B + \left(\left(1-\beta\right)k + \beta f_B\right)\frac{g_B}{M}\right). \end{aligned}$$

Note that k_s is independent of capital because an additional unit of capital, which translates into $1-\beta$ additional units of labor income, is paid to the bank before the second stage takes place in order to keep separation viable, and therefore it is not reinvested. However, $\frac{dk_p(k)}{dk} > 0$, because a pooling contract involves no early payment and therefore every additional unit of labor income is reinvested, augmenting current capital production. Invoking Lemma 3 we can then derive the transition function for capital as

$$k_{t+1} = \begin{cases} k_s & \text{if } k_t \leq \bar{k} \\ k_p(k_t) & \text{otherwise} \end{cases}. \tag{12}$$

How does k_s relate to $k_p(\bar{k})$? On the one hand, under separation, all of the productive entrepreneurs (entire measure μ) engage in production. Type B entrepreneurs do not apply for financing and do not crowd out type G from getting financed. On the other hand, to make separation possible, the income obtained in the first stage cannot be reinvested, so each type G entrepreneur produces less. In this paper, we focus on the set of parameters for which the composition effect dominates the per agent production effect at $\bar{k}$. We therefore make the following assumption.

Assumption 3 $k_p(\bar{k}) < k_s$.

This implies that as type B gets pooled into the mix, capital output goes down due to the crowding out of type G,[13] despite the fact that more resources are invested in production.

Also, we make an assumption to ensure there is no perpetual growth in this economy. We make sure that for pooling contracts, an extra unit of capital, which translates into an extra $1-\beta$ units of input into the second stage of production, results in less than one unit of additional capital.

Assumption 4 $\frac{dk_p(k)}{dk} = (1-\beta)\mu\left(\mu\frac{g_G}{M} + (1-\mu)\frac{g_B}{M}\right) < 1.$

In the following proposition we find restrictions on the set of parameters and initial condition k_0 that ensure existence of the certain type of dynamic equilibrium, in particular, the one that exhibits cyclical behavior. Note that we need to ensure that Assumption 2 holds for all admissible k.

Proposition 2 *Suppose Assumptions 1, 3, 4 hold. Suppose that $k_p(\bar{k}) < \bar{k} < k_s$. Consider $k_{min} := k_p(\bar{k})$ and $k_{max} := k_s$, and suppose that Assumption 2 holds for $\rho = \beta$ and $w = w(k_{max})$ in (a),(b),(d),(f) and $w = w(k_{min})$ in (c). Then for any $k_0 \in [k_{min}, k_{max}]$ there exists a dynamic equilibrium with the capital stock (and output) exhibiting cycles, not necessarily trivial. (Figure 4).*

Proof. See the appendix. ■

For cyclical equilibria,[14] the length of the cycle can be easily calculated from the primitives.

Corollary 1 *Consider an economy satisfying $k_p(\bar{k}) < \bar{k} < k_s$. If n is the smallest number such that $k_p^{(n-1)}(k_s) > \bar{k}$ but $k_p^n(k_s) < \bar{k}$, then an economy starting at $k_0 = k_s$ exhibits cycles of length $n+1$. In each cycle, the capital level declines for the first $n-1$ periods and goes up to k_s in the last.*

[13] Crowding out of type G caused by financing of type B entrepreneurs occurs because there is a limited supply of funds ($2M\mu$). Crowding out of type G, however, is not essential for our trend reversion mechanism. What would suffice is the existence of any technology, which is more productive than type B technology and financing of type B causing crowding out of funds away from that technology. For exposition purposes, however, we focus on the possibility of crowding out of funds from the most productive entrepreneurs.

[14] Note that it is also possible to derive parametric restrictions and bounds on k_0 that guarantee that $\bar{k} < k_p(\bar{k}) < k_s$ and existence of equilibrium such that the capital stock converges to $k_{ss} = k_p(\bar{k})$ (Figure 5). Finally, another possible equilibrium behavior is for the capital stock to converge to k_s. Such behavior would arise for parametric restrictions and bounds on k_0 that guarantee existence of equilibrium and ensure that $k_p(\bar{k}) < k_s < \bar{k}$.

Proof. See the appendix. ■

We give a numerical example, which demonstrates that the set of parameters and initial conditions satisfying restrictions of Proposition 2 is non-empty.

Example 1. Parameters are $\alpha = 0.32, \beta = 0.91,\ \mu = 0.64, f_G = 2.07, g_G = 2, f_B = 0, g_B = 1.6, R_f = 1, M = 1$. *With these parameters,* $\bar{k}$ *=4.58,* k_s *=5.02,* $k_p(k_s)$ *=4.6. Because these parameters imply* $k_p(k_s) > \bar{k}$*, we obtain non-trivial cycles, capital cycle given by 5.02, 4.6, 4.55, 5.02, 4.6, 4.55...*

5. Model with No externalities

We now consider the case with no externalities, that is $A_t := A$. The rental price of capital goods is no longer a constant; it depends on the belief of the the consumption good sector about the capital stock in the next period, which we denote by k'. Prices are given by $\rho(k') = A\beta k'^{\beta-1}$ and $w(k) = A(1-\beta)k^{\beta}$.

Although this introduces complications in the analysis of the model's dynamics, the reversion of the aggregate productivity arises according to the same intuition as discussed before. However, a new interesting phenomenon, an indeterminacy region, appears. For certain levels of capital k, two possible forecasts about future capital k' are self-fulfilling. If every representative consumption good firm believes that a separating contract will arise yielding high levels of capital holdings by the consumption good sector tomorrow, then currently produced capital will trade at a low price $\rho(k')$. This low price, in turn, will induce a separating contract, thus making the belief consistent with the equilibrium outcome. In the same way, if it believes that a pooling contract and hence lower levels of future capital holdings will arise, capital will trade at a high price. This price, in turn, will induce a pooling contract, making the belief consistent.

5.1. Prices, Equilibrium Contracts and Consistent Beliefs

As we discussed, the consumption good sector's belief regarding future capital holdings influences the behavior of entrepreneurs and equilibrium outcomes through its effect on the rental price of capital. We refer to (k, k') as the state of the economy, which determines equilibrium contracts and total production. Moreover, when looking for a dynamic equilibrium we require beliefs k' to be consistent with actual production.

We can write the minimum level of the early payment that ensures separation $\tilde{\delta}(w,\rho)$ and the interest rate $R(w,\rho,0)$ given in (1) and (6) as functions of (k,k'),

$$\begin{aligned} \tilde{\delta}(k,k') &= w(k) + \rho(k')f_B, && (13)\\ R(k,k',0) &= \frac{R_f}{\mu} - \frac{(1-\mu)}{2M\mu}\left[(1-\alpha)\rho(k')\left(g_B + \left(w(k)+\rho(k')f_B\right)\frac{g_B}{M}\right)\right]. && (14) \end{aligned}$$

As before, separating contracts emerge iff

$$\frac{1-\rho(k')\frac{g_G}{M}}{2M} \leq -\frac{R(k,k',0)-R_f}{\tilde{\delta}(k,k')}. \tag{15}$$

Note that as the belief k' increases, the price of capital $\rho(k')$ decreases, generating two important effects. First, an increase in the early payment δ now hurts type G less, as the benefits from extra reinvestment (due to the sale of extra units of capital in the second stage) decrease, and therefore type G's indifference curves (with slope $\frac{1-\rho(k')\frac{g_G}{M}}{2M}$) flatten in the $R-\delta$ plane. Second, the critical slope IPS ($-\frac{R(k,k',0)-R_f}{\delta(k,k')}$) steepens, because the minimum early payment that ensures separation declines, and the interest rate charged in case of pooling rises. As we can see, an increase in the belief k' has the same qualitative effect as a decline in k does, although for different reasons.

For a fixed k, there exists a cutoff level of beliefs, defined as the unique solution to (15) rewritten as equality, and denoted by $\tilde{k}'(k)$, which characterizes the equilibrium contracts. For beliefs k' below (above) this cutoff level, pooling (separating) contracts are selected. Hence, capital production, for given capital stock and belief, is

$$\kappa(k,k') = \begin{cases} \kappa_p(k,k') & \text{if } k' < \tilde{k}'(k), \\ \kappa_s(k') & \text{if } k' \geq \tilde{k}'(k) \end{cases} \tag{16}$$

where

$$\begin{aligned} \kappa_p(k,k') &= \mu^2[f_G + g_G + (w(k) + \rho(k')f_G)\frac{g_G}{M}] + (1-\mu)\mu[f_B + g_B + (w(k) + \rho(k')f_B)\frac{g_B}{M}], \quad (17) \\ \kappa_s(k') &= \mu[f_G + g_G + \rho(k')(f_G - f_B)\frac{g_G}{M}]. \quad (18) \end{aligned}$$

Figure 5 illustrates $\kappa(k,k')$ as a function of beliefs k' (for a fixed k). It consists of two segments: for low beliefs k', pooling arises, and for high beliefs k', separation arises. Both segments are downward sloping, as higher k' reduces reinvestment and hence total capital production. Moreover, for a given k', $\kappa_p(k,k')$ increases in k (due to higher reinvestments) and $\kappa_s(k')$ is independent of k (as the extra income is used for repayment). Finally, note that as k increases, a higher belief k' is needed to induce the selection of a separating contract. In fact, as k goes up the critical slope IPS flattens (as in Section 4), so it takes a higher belief k' (which implies a lower price $\rho(k')$ and therefore a steeper IPS and flatter indifference curves) to restore indifference between the two contracts for type G entrepreneurs. These facts are summarized in the following lemma.

Lemma 4 *The following are true*

$$\frac{\partial \kappa_p(k,k')}{\partial k} \geq 0, \frac{\partial \kappa_p(k,k')}{\partial k'} \leq 0, \frac{d\kappa_s(k')}{dk'} \leq 0, \frac{d\tilde{k}'(k)}{dk} \geq 0.$$

Proof. See the appendix. ■

We require that in accordance with the rational expectations hypothesis, equilibrium beliefs k' are consistent with equilibrium outcomes, i.e.,

$$k' = \kappa(k,k'). \tag{19}$$

The right hand side gives the future level of capital as a function of current capital k and beliefs about future capital k'. These beliefs, i.e., the left hand side of the equation, must equal the actual level of future capital. We define the consistent beliefs correspondence $k'(k)$ as the set of solutions to equation (19), i.e., the set of beliefs about future capital consistent with equilibrium outcomes when the current level is k. We now turn our attention to ensuring that this correspondence is non-empty, which is needed for equilibrium existence.

Note that because $\kappa_p(k, \cdot)$ and $\kappa_s(\cdot)$ are single-valued, the consistent belief correspondence can attain at most two values: a belief associated with pooling, and a belief associated with separation. We define these two candidate values for consistent beliefs below.

Definition 2 *Denote by k_s the solution to $\kappa_s(k') = k'$, and by $k_p(k)$ the solution (for fixed k) to $\kappa_p(k, k') = k'$ (both exist since $\kappa_s(\cdot)$ and $\kappa_p(k, \cdot)$ are decreasing).*

In order for belief k_s to be consistent, it must actually induce separation, that is, we must have $k_s \geq \tilde{k}'(k)$. Note that if this inequality is satisfied for some k, it is also satisfied for all smaller k, because $\tilde{k}'(k)$ is increasing in k. The maximum level of k satisfying this inequality is given by the solution to $\kappa_s(\tilde{k}'(k)) = \tilde{k}'(k)$, which we denote by k_{hi} (See Figure 6).[15] Hence, the belief k_s is consistent for all $k \in [0, k_{hi}]$.

In order for belief $k_p(k)$ to be consistent, it must induce pooling, that is, we must have $k_p(k) \leq \tilde{k}'(k)$. We define the minimum level of capital for which $k_p(k)$ is consistent as $k_{li} = \inf\{k | k_p(k) \leq \tilde{k}'(k)\}$ (See Figure 7.). If the set is empty, $k_{li} = +\infty$.

In order to ensure the existence of equilibrium, a consistent belief must exist for every capital level k in the equilibrium path. We already showed that for for $k \leq k_{hi}$, there is at least one consistent belief (k_s). We will now seek the restrictions needed to guarantee the existence of a consistent belief for $k > k_{hi}$. We do this in two steps. First, we guarantee that $k_{li} < k_{hi}$. Second, we guarantee that $k_p(k)$ is consistent for $k > k_{li}$.

The next assumption guarantees that $k_p(k)$ is consistent for $k = k_{hi}$ and therefore that $k_{li} \leq k_{hi}$ (see Lemma 5 and Figure 6).

Assumption 5 $\tilde{k}'(k_{hi}) > \kappa_p(k_{hi}, \tilde{k}'(k_{hi})) > 0$.

Lemma 5 *If Assumption 5 is satisfied, then $k_{hi} \in \{k | k_p(k) \leq \tilde{k}'(k)\}$ and therefore $k_{li} \leq k_{hi}$.*

Proof. By Assumption 5, $\kappa_p(k_{hi}, \tilde{k}'(k_{hi})) < \tilde{k}'(k_{hi})$. Since $\kappa_p(k_{hi}, 0^+) > 0$, the continuity of $\kappa_p(k_{hi}, \cdot)$ implies the existence of $k_p(k_{hi}) \in (0, \tilde{k}'(k_{hi}))$. We then conclude that $k_{hi} \in \{k | k_p(k) \leq \tilde{k}'(k)\}$. Recalling the definition of k_{li}, we have $k_{li} \leq k_{hi}$. ■

The next assumption, made parametric in Proposition 3, ensures that $k_p(k)$ is consistent for $k \geq k_{li}$ (See Lemma 6). Note that this assumption is more restrictive than what is necessary to ensure existence of equilibrium, for which consistency of $k_p(k)$ for $k > k_{hi}$. We choose to be more restrictive to have a more tractable structure of the consistent belief correspondence.

Assumption 6 $\frac{d}{dk}[k_p(k) - \tilde{k}'(k)] \leq 0$ *for $k \geq k_{li}$.*

Lemma 6 *If Assumptions 5 and 6 are satisfied for all $k \geq k_{li}$, there exists a consistent pooling belief.*

Proof. The belief about pooling is consistent as long as $k_p(k) - \tilde{k}'(k) \leq 0$. By Lemma 5, k_{li} exists. Moreover, this inequality is satisfied for k_{li} by definition of k_{li}. Assumption 6 then guarantees the result. ■

[15] If the solution does not exist, let $k_{hi} = 0$.

Lemma 4 allows us to deduce the shape of the consistent belief correspondence $k'(k)$:

$$k'(k) = \begin{cases} k_s & \text{if } k < k_{li} \\ \{k_s, k_p(k)\} & \text{if } k \in [k_{li}, k_{hi}] \\ k_p(k) & \text{if } k > k_{hi} \end{cases}, \tag{20}$$

For low levels of capital, a belief associated with separation is the unique consistent belief. For intermediate levels of capital, a belief associated with separation and a belief associated with pooling are both consistent. We refer to this range of capital levels as the indeterminacy region. Finally, for high levels of capital, a belief associated with pooling is the unique consistent belief (See Figures 8-10.).

The transition function for capital is then

$$k_{t+1} = \begin{cases} k_s & \text{if } k_t < k_{li} \text{ or } (k_t \in [k_{li}, k_{hi}] \text{ and } k'_t = k_s) \\ k_p(k_t) & \text{if } k_t > k_{hi} \text{ or } (k_t \in [k_{li}, k_{hi}] \text{ and } k'_t = k_p(k_t)) \end{cases}. \tag{21}$$

5.2. Dynamic Equilibrium

The consistent belief correspondence and the transition function, derived above, allow us to characterize the dynamic behavior of the capital stock, aggregate productivity and total output in the economy. The next assumption, made parametric in Proposition 3, corresponds to Assumption 4 of Section IV, and is sufficient to guarantee that there is no perpetual growth.

Assumption 7 $\frac{\partial \kappa_p(k,k')}{\partial k} < 1$.

Figure 11 illustrates one possible transition correspondence. Assumption 7 together with Lemma 4 imply that $\frac{dk_p(k)}{dk} \in [0,1]$.[16] Assumption 5 implies $k_p(k_{hi}) < k_s$.

In the following two propositions, we find restrictions on the set of parameters and the initial conditions in order to guarantee the existence of dynamic equilibria that may exhibit cyclical behavior. In Proposition 3, cycles always exist, but the selection of beliefs in the indeterminacy region dictates their length and amplitude. In Proposition 4, the selection of beliefs dictates whether the equilibrium exhibits cycles or convergence to a steady state. Note that Assumptions 2, 6, 7 depend on k and k'. The hypothesis of the propositions must ensure that these assumptions hold along the equilibrium paths.

Proposition 3 *Suppose Assumptions 1 and 5 are satisfied and $k_p(k_{li}) \leq k_{li} \leq k_s$ and $k_p(k_{hi}) \leq k_{hi} \leq k_s$. Consider $k_{min} := k'_{min} := k_p(k_{li})$ and $k_{max} := k'_{max} = k_s$ and suppose that Assumption 2 holds for $\rho = \rho(k'_{max})$ in (a), $\rho = \rho(k'_{min})$ and $w = w(k_{max})$ in (b), $\rho = \rho(k_p(k_{max}))$ and $w = w(k_{max})$ in (d), $\rho = \rho(k'_{min})$ in (e) and (f). For (c), assume that if (2) is satisfied for $\rho = \rho(k'_{min})$, then (3) holds for $\rho = \rho(k_p(k_{max}))$ and $w = w(k_{min})$. If (2) fails for $\rho = \rho(k'_{max})$, then (4) holds for $\rho = \rho(k_p(k_{max}))$ and $w = w(k_{min})$. If neither is true, then both (3) and (4) hold for $\rho = \rho(k_p(k_{max}))$ and $w = w(k_{min})$. Finally, suppose that $\max_{[k_{li},k_{max}]} \frac{d}{dk}[k_p(k) - \tilde{k}'(k)] \leq 0$ (for Assumption 6 to hold) and $[\mu^2 \frac{g_G}{M} + (1-\mu)\mu \frac{g_B}{M}]w'(k_{max}) < 1$ (for Assumption 7 to hold).*

[16] Indeed, implicitly differentiating $\kappa_p(k, k_p(k)) = k_p(k)$ gives $\frac{dk_p(k)}{dk} = \frac{d\kappa_p(k,k')}{dk} / \left(1 - \frac{d\kappa_p(k,k')}{dk'}\right) \in [0,1]$.

Then for any $k_0 \in [k_{min}, k_{max}]$ defined in (20), there exists a dynamic equilibrium with the capital stock (and output) exhibiting perpetual cycles regardless of the selection from the correspondence $k'(\cdot)$. However, if the selection is "optimistic" (i.e. $k'(k) = k_s$ for $k \in [k_{li}, k_{hi}]$), the cycles are shorter and smaller in amplitude.

Proof. See the appendix. ■

Proposition 4 *Suppose the hypothesis of Proposition 4 holds except that $k_p(k_{li}) \leq k_{li} \leq k_s$ and $k_{hi} \geq k_s$. Then, for any $k_0 \in [k_{min}, k_{max}]$, equilibrium exists, and its characteristics depend on the selection from the correspondence $k'(\cdot)$. If the selection is optimistic, there are no cycles, both k and k' converging to k_s. However, if the consumption good sector is pessimistic, the economy exhibits perpetual cycles.*

Proof. See the appendix. ■

We give a numerical example, which shows the restrictions derived on the set of parameters and initial conditions satisfying the restrictions in Proposition 3 are non-empty and cyclical behavior is possible.

Example 2. Consider parameters $A = 2.66, \alpha = 0.82, \beta = 0.89, \mu = 0.33, f_G = 2, g_G = 2, f_B = 0, g_B = 1.6, R_f = 1, M = 1$. With these parameters, $k_s = 4.06$, $k_{li} = 2.316$,$k_{hi} = 2.63$, $k_p(k_s) = 2.327$. Because these parameters imply $k_p(k_s) > k_{li}$, we obtain a three-period cycle ($k = 4.06$, 2.301, 2.114, 4.06...) as long as the selection of beliefs in the indeterminacy region is pessimistic. The dynamics for this case are illustrated in Figure 12. If the selection of beliefs for k in the indeterminacy region is optimistic, then the two-period cycle emerges ($k = 4.06$, 2.301, 4.06...).

6. Model with Endogenous Savings

In our previous setup, entrepreneurs saved a part of their end of period net worth at the risk-free rate in international markets. Their net worth depended on the current level of capital in the economy, but we assumed that the amount of loanable funds available for financing the young entrepreneurs of each generation was fixed at the level of $2M\mu$. In this section, we extend the analysis of the case presented in Section IV by endogenizing the supply of loanable funds. In particular, we require that loans to the young entrepreneurs are financed with the old entrepreneurs' savings. To keep the analysis tractable, we keep the risk-free interest rate fixed as in the case of a small open economy.

Endogenizing the supply of loanable funds not only confirms the possibility of cyclical economic behavior, but it also gives rise to the possibility of sudden drops and slow recoveries, which have been widely documented in the literature. This extended model also generates predictions that are qualitatively consistent with much of the empirical evidence: procyclicality of net worth, cash flows, investment and loanable funds, lower reliance on bank financing at the top of the cycle and default rates lagging after the business cycle.

The intuition for slow expansions and sudden recessions is as follows. For the same reasons as those discussed in Section IV, for low enough levels of capital, separating contracts emerge and only type G entrepreneurs enter production. However, if the supply of funds is also low, only a small fraction of type G entrepreneurs is financed. Aggregate productivity and capital production increase from one period to the next, and so do the savings. As savings increase, a higher fraction of type G entrepreneurs is financed and the recovery continues until capital reaches a high enough level that induces the selection of pooling contracts. As pooling emerges, however, high levels of funds, which increase individual production, may not be enough to offset the decline in productivity generated by the mix of entrepreneurs engaged in production. Hence, in contrast to the previous setup, endogenous supply of funds can give rise to cyclical dynamic equilibria exhibiting slow expansions and sudden recessions. Even though we focus on the case of slow recoveries and sudden drops, which we find particularly interesting, endogenizing the evolution of loanable funds in general allows for a richer cyclical dynamics.

In what follows, we derive the dynamical system describing the evolution of the two state variables (capital and savings). We then analyze their behavior using the phase diagram and illustrate the possibility of cyclical dynamic equilibria exhibiting sudden drops and slow recoveries.

6.1. Transition Functions

The funds used to finance the young generation are given by the savings S_t of the old generation. If funds are not sufficient to finance all applicants, crowding out occurs, with the unfinanced entrepreneurs staying out of capital production and obtaining only $w(k)$ as labor income. If funds are in excess of applicants' demand, every applicant gets financed. The excess funds are saved in international markets at the risk-free rate R_f.

It follows from the discussion in Section IV that whether pooling or separating contracts emerge depends on k (according to Lemma 3) and not on S. The amount of savings S, however, is important as it affects the measure of entrepreneurs financed and influences capital and loanable funds in the next period. In case of a pooling contract, the available funds are used to finance both type G and type B entrepreneurs, the total demand for funds is $2M$. In case of a separating contract, only type G is financed, and the total demand for funds equals to $2M\mu$. Capital stock in period $t+1$, given the current state variables k_t, S_t, is then

$$k_{t+1}(S_t, k_t) = \begin{cases} \frac{S_t}{2M}\left[\mu k_p^G(k_t) + (1-\mu)k_p^B(k_t)\right] & \text{if } k_t > \bar{k} \text{ and } S_t \leq 2M \\ \mu k_p^G(k_t) + (1-\mu)k_p^B(k_t) & \text{if } k_t > \bar{k} \text{ and } S_t > 2M \\ \frac{S_t}{2M}k_s^G & \text{if } k_t \leq \bar{k} \text{ and } S_t \leq 2M\mu \\ \mu k_s^G & \text{if } k_t \leq \bar{k} \text{ and } S_t > 2M\mu \end{cases}, \tag{22}$$

where

$$\begin{aligned} k_s^G &= f_G + g_G + \beta\left(f_G - f_B\right)\frac{g_G}{M}, \\ k_p^i(k_t) &= f_i + g_i + \left(\left(1-\beta\right)k_t + \beta f_i\right)\frac{g_i}{M}, i = G, B, \end{aligned}$$

denote individual capital production of a type G entrepreneur under separation and of each type under pooling respectively.

Homotheticity of preferences and constancy of R_f imply that entrepreneurs always save a constant fraction ($:= \xi$) of their end of period net worth. Note that the unfinanced entrepreneurs, whether they are type B due to exclusion under separation or both types due to crowding out under pooling, save fraction ξ of the wage income $(1-\beta)\,k$. We obtain the supply of funds tomorrow as a function of the current state variables:

$$S_{t+1}(S_t,k_t,)=\begin{cases}\xi\left[\frac{S_t}{2M}\left(\mu W_p^G(k_t)+(1-\mu)W_p^B(k_t)\right)+\left(1-\frac{S_t}{2M}\right)(1-\beta)k_t\right] & \text{if } k_t>\bar{k} \text{ and } S_t\leq 2M\\ \xi\left[\mu W_p^G(k_t)+(1-\mu)W_p^B(k_t)\right] & \text{if } k_t>\bar{k} \text{ and } S_t>2M\\ \xi\left[\frac{S_t}{2M}W_s^G+\left(1-\frac{S_t}{2M}\right)(1-\beta)k_t\right] & \text{if } k_t\leq\bar{k} \text{ and } S_t\leq 2M\mu\\ \xi\left[\mu W_s^G+(1-\mu)(1-\beta)k_t\right] & \text{if } k_t\leq\bar{k} \text{ and } S_t>2M\mu\end{cases}, \tag{23}$$

where

$$\begin{aligned}W_s^G &= \beta[g_G+\beta(f_G-f_B)\frac{g_G}{M}]-2MR_f,\\ W_p^G(k_t) &= \beta\left[g_G+((1-\beta)k_t+\beta f_G)\frac{g_G}{M}\right]-\frac{2MR_f}{\mu}+\frac{1-\mu}{\mu}(1-\alpha)\beta\left[g_B+((1-\beta)k_t+\beta f_B)\frac{g_B}{M}\right],\\ W_p^B(k_t) &= \alpha\beta\left[g_B+((1-\beta)k_t+\beta f_B)\frac{g_B}{M}\right]\end{aligned}$$

represent the net worth of type G entrepreneur under separation, type G entrepreneur under pooling, and type B entrepreneur under pooling, respectively.

6.2. The Phase Diagram

As usual for the analysis of such a dynamical system, we now divide the state space (S_t, k_t) into regions where it is possible to sign the changes $k_{t+1}-k_t$ and $S_{t+1}-S_t$. Because only k matters for whether separating or pooling contracts emerge, the horizontal line given by $k=\bar{k}$ splits the state space (S_t, k_t) into the region of separating contracts (the region below the line) and the region of pooling contracts (the region above the line). Our analysis is carried out separately for these two regions.

First, consider the region of separation ($k_t\leq\bar{k}$). From (22) and (23), we derive that $k_{t+1}=k_t$ iff

$$k_t=\begin{cases}\frac{S_t}{2M}k_s^G & \text{if } S_t\leq 2M\mu\\ \mu k_s^G & \text{if } S_t>2M\mu\end{cases}, \tag{24}$$

and $S_{t+1}=S_t$ iff

$$k_t=\begin{cases}\frac{(2M/\xi-W_G^s)S_t}{(1-\beta)(2M-S_t)} & \text{if } S_t\leq 2M\mu\\ \frac{S_t/\xi-\mu W_G^s}{(1-\beta)(1-\mu)} & \text{if } S_t>2M\mu\end{cases}. \tag{25}$$

Note that (24) is an upward sloping linear curve for $S_t\leq 2M\mu$, at which point it connects to a horizontal line. Moreover, $k_{t+1}-k_t<0$ for points above and to the left of this curve, and $k_{t+1}-k_t>0$ otherwise. The expression in (25) is a hyperbola in the range $S_t\leq 2M\mu$, connecting at $S_t=2M\mu$ to an upward sloping line. Moreover, $S_{t+1}-S_t>0$ above the resulting curve and $S_{t+1}-S_t<0$ below it.

Second, we consider the region of pooling ($k_t>\bar{k}$). From (22) and (23), we obtain that $k_{t+1}=k_t$ iff

$$k_t=\begin{cases}\frac{A'S_t}{2M-B'S_t} & \text{if } S_t\leq 2M\\ \frac{A'}{1-B'} & \text{if } S_t>2M\end{cases}, \tag{26}$$

where $A'=\mu\left(f_G+g_G+\beta f_G\frac{g_G}{M}\right)+(1-\mu)\left(f_B+g_B+\beta f_B\frac{g_B}{M}\right)$ and $B'=\mu(1-\beta)\frac{g_G}{M}+(1-\mu)(1-\beta)\frac{g_B}{M}$, and $S_{t+1} = S_t$ iff

$$k_t = \begin{cases} \frac{(2M/\xi - A)S_t}{BS_t + (2M - S_t)(1-\beta)} & \text{if } S_t \leq 2M \\ \frac{S_t/\xi - A}{B} & \text{if } S_t > 2M \end{cases}, \tag{27}$$

where $A = \beta\left[\mu\left(g_G + \frac{\beta f_G g_G}{M}\right) + (1-\mu)(g_B + \frac{\beta f_B g_B}{M})\right] - 2MR_f$ and $B = \beta(1-\beta)(\mu\frac{g_G}{M} + (1-\mu)\frac{g_B}{M})$.

Assuming that $B' < 1$ to avoid perpetual growth,[17] we have that (26) is a positive hyperbola connecting to a horizontal line at $S_t = 2M$. In this case, $k_{t+1} - k_t < 0$ to the left and above of the curve and $k_{t+1} - k_t > 0$ below and to the right of the curve. The curve given by (27) in the range $S_t \leq 2M$ connects to a positively sloping line at $S_t = 2M$. We have $S_{t+1} - S_t > 0$ above and $S_{t+1} - S_t < 0$ below the resulting curve.

Note that the definition of the dynamic equilibrium (Definition 1) must be modified for the extended environment. In particular, the equilibrium path of $\{S_t^*\}_{t=0}^{\infty}$ must be specified. While the prices and contracts are determined as previously, k_t^* and S_t^* must now evolve according to (22) and (23). Note that prices must still satisfy Assumption 2. Our goal is to illustrate an example of a cyclical dynamic equilibrium that exhibits long expansions and short but severe recessions.

By setting the restrictions below, we essentially focus on a particular configuration of the phase diagram, one that allows for the possibility of equilibrium paths that exhibit slow expansions and sudden drops.

Assumption 8 *Suppose that $\mu k_s^G > \bar{k}$ and $B' < 1$. In that case, the solution to (24) at $k = \bar{k}$ is given by $S_1 = \frac{2M\bar{k}}{k_s^G}$, and the solution to (26) at $k = \bar{k}$ is given by $S_3 = \frac{2M\bar{k}}{A' + B'\bar{k}}$. Denote by S_2 and S_4 the respective solutions to (25) and (27) at $k = \bar{k}$. Suppose that $S_4 < S_1 < \min\{S_2, S_3\}$.*

Assuming that $S_1 < S_3$ is equivalent to the parametric restriction we made in Assumption 3. Assuming $S_4 < S_1$ ensures that the equilibrium path does not converge to a steady state with separating contracts, while $S_1 < S_2$ assumes away the convergence to a steady state with pooling contracts.

We find the parametric example satisfying Assumption 8 that gives rise to the cyclical dynamic equilibrium exhibiting sudden drops and slow recoveries.

Example 3. Suppose that $\xi = 0.75, \alpha = 0.44, \beta = 0.975, \mu = 0.26, f_G = 2.41, g_G = 1.83, f_B = 0.77, g_B = 1.825, R_f = 1, M = 1$. There exists a cyclical dynamic equilibrium for $(S_0, k_0) = (0.4, 1.2)$ exhibiting 11-period cycles with slow recoveries and sudden drops ($s_t =$ 0.35, 0.362, 0.374, 0.385, 0.397, 0.41, 0.422, 0.435, 0.448, 0.46, ...0.35... and $k_t =$ 1.197, 1.26, 1.29, 1.34, 1.38, 1.42, 1.46, 1.51, 1.55, 1.6003, ... 1.197...). Capital and supply of funds rise slowly and drop suddenly. The drop occurs immediately upon k exceeding *$\bar{k}$. (See Figures 13 a and b.)*

In the example given above, capital and supply of funds are low at the trough. Separating contracts emerges, but as the demand for funds ($= \mu 2M$) exceeds the supply, even type G entrepreneurs are rationed. Aggregate productivity, capital and savings rise. With more loanable funds available more type G entrepreneurs enter production and the expansion continues. Because the expansion is very slow, the peak level of capital is just slightly above $\bar{k} = 1.6$. At the peak, capital is high enough to warrant pooling.

[17] This is slightly stronger than Assumption 4, as it ensures no perpetual growth with all entrepreneurs entering production under pooling, not only a fraction μ of them. If $B' > 1$, then the curve is negative in the range $S_t \in (2M/B', 2M)$, connecting to a zero-sloping line at $S_t = 2M$. In that case, $k_{t+1} - k_t < 0$ to the left of the convex curve in range $S_t \in (0, 2M/B')$ and $k_{t+1} - k_t > 0$ for all the points to its right in the first quadrant, thus allowing for perpetual growth in that region.

However, the demand for funds ($= 2M$) still exceeds the supply, and there is crowding out of type G entrepreneurs by type B entrepreneurs, thus setting off a recession.

Because we construct the proposed equilibrium path using the appropriate contract determination and the appropriate evolution of capital and savings, to prove that the resulting paths comprise the dynamic equilibrium, it remains to check that the implied prices satisfy the conditions of Assumption 2. These can be easily checked using the equilibrium price definitions and the provided values of $\{k_t^*\}$.

In the example economy, capital, savings, investment, loanable funds, output, cash flows and net worth move together. These variables are procyclical. Moreover, because of procyclicality of cash flows (labor income and revenues from capital sales after the first stage), the reliance on banks for financing of second stage production (and hence overall production) declines during the expansions and rises at troughs. Finally, the default rates are highest following the peak. Hence, our extended model generates predictions that are qualitatively consistent with much of the empirical evidence.

7. Conclusion

Recent empirical work has suggested that the systematic behavior of lending standards over the business cycle, with laxer standards applied during expansions and tighter standards applied during recessions is responsible for the reversion of trends in aggregate productivity. We propose a novel productivity reversion mechanism that rationalizes these empirical findings.

We build a dynamic screening model with fully rational entrepreneurs of privately known types and a competitive lending sector. Entrepreneurs must seek financing of their projects from the lending sector. Screening is done with an early payment on the loan. The equilibrium contract menus are found as the best for the productive entrepreneurs subject to the zero lenders' profits.

As the (endogenously evolving) productivity rises, liquidity is high for all types of producers, allowing even the unproductive type to meet the early payments on the loan. The early payment required to accomplish screening out the unproductive types thus rises. Because the early payment hurts productive entrepreneurs by lowering their investments, a separating contract menu becomes less attractive for them. Moreover, a pooling menu (with zero early payments) become more attractive, as it calls for a lower cross-subsidy in periods with high liquidity. Hence, low productivity entrepreneurs enter production along with productive types, the composition effect setting off a recession. The opposite happens in periods with low enough productivity. Thus, aggregate productivity over this cycle changes due to the composition of types of projects being financed and implemented.

When the supply of loanable funds is endogenized, we find the possibility of much richer cyclical dynamics and predictions that are qualitatively consistent with much of the empirical evidence: procyclicality of net worth, cash flows, investment and loanable funds, lower reliance on bank financing at the top of the cycle, default rates lagging after the business cycle, and finally, the possibility of slow recoveries and abrupt and severe recessions.

Future research can benefit from embedding our reversion mechanism within the general equilibrium model with infinitely lived agents and assessing its importance quantitatively.

8. Appendix

Credit Line as the Alternative Form of Contracts

Identical equilibrium outcomes are achieved under the assumption of contracts in the form of a credit line with the credit limit $2M - \delta$ and interest rate $\frac{2MR-\delta}{2M-\delta}$.

In the first stage the entrepreneur borrows M and starts production (he does not borrow more since it would not be productive to do so). In the second stage he taps the credit line for the remaining $M - \delta$, which could be complemented with his own savings, so his disposable income is $M - \delta + \rho f_i + w$, exactly as in the paper. Finally, he would have to repay $2MR - \delta$ in the last stage, so the effective interest rate is $\frac{2MR-\delta}{2M-\delta}$.

Note that in this case the only requirement is that the bank can monitor that production is started in the second stage, which in the case of a bad entrepreneur requires he puts at least δ extra towards the project.

Proof of Lemma 1

▶ By Assumption 1, it is enough to prove that type B is willing to reinvest. One extra unit of consumption good reinvested into the second stage of production yields g_B/M units of capital, generating $\rho g_B/M$ units of the consumption good. Therefore, for reinvestment to be profitable, we must have $\rho g_B/M > 1$, which is true by condition (a).

▶ We now show that type B enters and defaults whenever offered a contract with $\delta_B < \tilde{\delta}(w, \rho)$ and $R_B \geq R_f$. First, note that repaying is dominated by defaulting whenever

$$\alpha\rho[g_B + (w + \rho f_B - \delta_B)\frac{g_B}{M}] \geq \rho[g_B + (w + \rho f_B - \delta_B)\frac{g_B}{M}] - (2MR_B - \delta_B).$$

Noting that the most restrictive case is for $R_B = R_f$ and $\delta_B = \tilde{\delta}(w, \rho)$ (this last fact is due to condition (e)), the above inequality is implied by

$$\begin{aligned} \alpha\rho[g_B + (w + \rho f_B - \tilde{\delta}(w, \rho))\frac{g_B}{M}] &\geq \rho[g_B + (w + \rho f_B - \tilde{\delta}(w, \rho))\frac{g_B}{M}] - \left(2MR_f - \tilde{\delta}(w, \rho)\right), \text{ i.e.,} \\ \alpha\rho g_B &\geq \rho g_B - 2MR_f + w + \rho f_B, \text{ i.e.,} \\ w + \rho f_B + (1-\alpha)\rho g_B &\leq 2MR_f, \end{aligned}$$

which is true by condition (b).

Second, note that type B prefers to enter (and default) rather than reject the contract whenever for any $\delta_B < \rho f_B + w$,

$$\alpha\rho[g_B + (w + \rho f_B - \delta_B)\frac{g_B}{M}] \geq w,$$

which is implied by condition (d).

▶ Fix an arbitrary $\delta_B < \tilde{\delta}(w, \rho)$. We want to show that there exists an $\hat{R}$ such that a pooling menu $((\delta_B, \hat{R}), (\delta_B, \hat{R}))$ is in F. Note that any $R > R_f$ would induce type B to enter and default (as shown above). Consider an $\hat{R}$ constructed to yield exactly a zero profit to the bank under the assumption that type G enters and repays:[18]

$$\hat{R} = R(w, \rho, \delta_B) = \frac{R_f}{\mu} - \frac{(1-\mu)}{2M\mu}\left[\delta_B + (1-\alpha)\rho\left(g_B + (w + \rho f_B - \delta_B)\frac{g_B}{M}\right)\right]. \tag{28}$$

Next we will show that $\hat{R} > R_f$, i.e. that the contract $\left(\delta_B, \hat{R}\right)$ requires cross-subsidization from type G. Note that this would be implied by

$$(1-\alpha)\rho\left[g_B + (w + \rho f_B - \delta_B)\frac{g_B}{M}\right] + \delta_B < 2MR_f, \tag{29}$$

i.e. if the bank collected less from defaulting type B than the opportunity cost of funds. The derivative of the left hand side with respect to δ is given by $1 - (1-\alpha)\rho g_B/M$, which is positive by condition (e). Hence, if (29) holds for $\delta_B = \tilde{\delta}(w, \rho)$, which is guaranteed by condition (f), it holds for all $\delta_B < \tilde{\delta}(w, \rho)$. So, we have $\hat{R} = R(w, \rho, \delta_B) > R_f$.

In order to prove that $((\delta_B, R(w, \rho, \delta_B)), (\delta_B, R(w, \rho, \delta_B)))$ is in F, it remains to show that type G indeed enters and repays.

First, note that type G prefers repaying to defaulting whenever

$$\begin{aligned}
\alpha\rho[g_G + (w + \rho f_G - \delta_B)\frac{g_G}{M}] &\leq \rho[g_G + (w + \rho f_G - \delta_B)\frac{g_G}{M}] - (2MR(w, \rho, \delta_B) - \delta_B), \text{ i.e.,} \\
(1-\alpha)\rho[g_G + (w + \rho f_G - \delta_B)\frac{g_G}{M}] &\geq 2MR(w, \rho, \delta_B) - \delta_B, \text{ i.e.,} \\
(1-\alpha)\rho[g_G + (w + \rho f_G)\frac{g_G}{M}] &\geq (1-\alpha)\rho\delta_B\frac{g_G}{M} + 2MR(w, \rho, \delta_B) - \delta_B. \qquad (30)
\end{aligned}$$

Because the derivative of the right hand side with respect to δ_B is given by

$$\mu((1-\alpha)\frac{\rho g_G}{M} - 1) + (1-\mu)((1-\alpha)\frac{\rho g_B}{M} - 1),$$

there are two possible cases.

Case 1: Suppose $\mu((1-\alpha)\frac{\rho g_G}{M} - 1) + (1-\mu)((1-\alpha)\frac{\rho g_B}{M} - 1) \leq 0$. Then inequality (30) is most difficult to satisfy for $\delta = 0$ and the corresponding $R(w, \rho, 0)$ defined above, in which case the inequality simplifies to

$$2MR_f \leq (1-\alpha)\rho\left[\mu[g_G + (w + \rho f_G)\frac{g_G}{M}] + (1-\mu)[g_B + (w + \rho f_B)\frac{g_B}{M}]\right].$$

This is implied by (3).

Case 2: Suppose $\mu((1-\alpha)\frac{\rho g_G}{M} - 1) + (1-\mu)((1-\alpha)\frac{\rho g_B}{M} - 1) \geq 0$. Then inequality (30) is most difficult to satisfy for $\delta \longrightarrow \tilde{\delta}$ and the corresponding $R(w, \rho, \delta)$, in which case the inequality simplifies to

$$2MR_f \leq \mu\left[(1-\alpha)\rho[g_G + \beta(f_G - f_B)\frac{g_G}{M}] + \rho f_B + w\right] + (1-\mu)\left[\rho f_B + w + (1-\alpha)\beta g_B\right].$$

This is implied by (4). Hence, condition (c) implies that G prefers to repay in full if he enters the

[18] Under such pooling contract (R, δ_B), both types enter, type G repaying in full while type B defaulting. Because types are unobservable and only $2M\mu$ can be loaned out, a measure μ^2 of entrepreneurs of type G and a measure $(1-\mu)\mu$ of type B are financed. Hence, the zero profit condition is given by $\mu^2 2MR + (1-\mu)\mu\left[[\delta_B + (1-\alpha)\rho(g_B + (w + \rho f_B - \delta_B)\frac{g_B}{M})]\right] = \mu 2MR_f$, which implicitly defines $R(w, \rho, \delta_B)$ given in (28).

pooling contract constructed here.

Next, we show that type G enters this contract. As already shown, type G prefers repaying to defaulting under the pooling contract. Since his utility of default is larger than type B's utility of default (due to Assumption 1), and since type B prefers entering and defaulting rather than working for w, we conclude without extra assumptions that type G enters the pooling contract (and repays).

Finally, any menu in F that induces type G to repay must satisfy $R_G \geq \hat{R}$ (δ_G is irrelevant in this case for the bank collection) with positive profits if $R_G > \hat{R}$ by the definition of $\hat{R}$. If it induces type G to default it must be the case that $\delta_G = \delta_B$ (by incentive compatibility) and that $R_G > \hat{R}$, since interest rate makes the repayment option less attractive, and type G repays at the contract $(\delta_G, \hat{R})$. In such a case, from the point of view of the bank, default is equivalent to type G defaulting under the contract $(\delta_G, \hat{R})$, which gives the bank more profit than type G repaying (that's the reason type G chooses to repay in that case). Since the last contract gives zero profits to the bank, the result follows.

► We show that $((\tilde{\delta}(w,\rho), R_f), (\tilde{\delta}(w,\rho), R_f))$ is in F. Type B cannot afford the contract and rejects it. From the previous part, and by continuity, we know that type G decides to enter and repay when offered a contract $(\tilde{\delta}(w,\rho), R(w,\rho,\tilde{\delta}(w,\rho)))$. Replacing the interest rate by $R_f < R(w,\rho,\tilde{\delta}(w,\rho))$ makes the repayment option more attractive without changing the default or non-entry options, hence type G will enter and repay in full when offered $(\tilde{\delta}(w,\rho), R_f)$.

Proof of Lemma 2

From Lemma 1 and the zero profit condition, we know that if $\delta_B < \tilde{\delta}(w,\rho)$, the only possible menus are in the set $\{((\delta_B, R_B), (\delta_G, R_G)) | \delta_G \geq \delta_B, R_B \geq \hat{R}\}$. WLOG, we can consider $R_B = \hat{R}$, since the interest rate for type B entrepreneurs is irrelevant for both the bank and the entrepreneurs themselves. Moreover, any menu $((\delta_B, \hat{R}), (\delta_G, \hat{R}))$ with $\delta_G > \delta_B$ is strictly dominated, for type G entrepreneurs, by $(((\delta_B, \hat{R}), (\delta_B, \hat{R})))$, therefore it is never offered in equilibrium.

Proof of Proposition 2

Consider $\{k_t^*\}_{t\geq 0}$ defined inductively as $k_0^* = k_0$, and for a given k_t^*, define k_{t+1}^* as in (12). Let $k_{min}^* := \inf_t k_t^*$ and $k_{max}^* := \sup_t k_t^*$. Define a sequence of beliefs $\{k_t'^*\}_{t\geq 0}$ by setting $k_t'^* = k_{t+1}^*\ \forall t$. Define price sequences by $w_t^* = (1-\beta)k_t^*$ and $\rho_t^* = \beta$. Given these prices, define (R_t^*, δ_t^*) as in Proposition 1. It remains to check that conditions of Assumption 2 are satisfied for ρ_t^* and w_t^* for all t. Noting that $w(k)$ is increasing in k, the hypothesis guarantees exactly this as long as $k_{min}^* \geq k_p(\bar{k})$ and $k_{max}^* \leq k_s$. We prove this by showing inductively that the entire sequence $\{k_t^*\}_{t\geq 0}$ lies within $[k_p(\bar{k}), k_s]$.

We have $k_p(\bar{k}) \leq k_0^* \leq k_s$ by hypothesis. Suppose that $k_p(\bar{k}) \leq k_t^* \leq k_s$. Then $k_{t+1}^* \geq \min\{k_s, k_p(k^*)\} \geq \min\{k_s, k_p(\bar{k})\} \geq k_p(\bar{k})$, where the second inequality is due to $\frac{dk_p(k)}{dk} > 0$ and the last one is due to the hypothesis that $k_p(\bar{k}) \leq k_s$. Also, $k_{t+1}^* \leq \max\{k_s, k_p(k_t^*)\} \leq \max\{k_s, k_t^*\} \leq k_s$ where the second inequality is implied by Assumption 4 and hypothesis $k_p(\bar{k}) \leq k_t^*$, while the last inequality is due to hypothesis that $k_t^* \leq k_s$.

Now, to see the existence of cycles, consider $k_0 < \bar{k}$ (the other case is analogous). Then $k_1 = k_s > \bar{k}$. It remains to show that $\exists N$ such that $k_p^N(k_1) < \bar{k}$. Since $k_p(k)$ is a linear function, we can write $k_p(k) = a + bk$, with $b < 1$ by Assumption 4. This, together with Assumption 3, implies that $k_1 - k_p(k) > 0$.

Then $k_n - k_{n+1} = b^n \left[(1-b)\, k_1 - a\right] = b^n \left[k_1 - k_p\left(k_1\right)\right] \to 0^+$. Since by hypothesis $\bar{k} - k_p\left(\bar{k}\right) > 0$, $\exists N$ such that $k_p^N(k_1) < \bar{k}$. Then $k_{N+1} = k_s$ and the cycle repeats itself.

Proof of Corollary 1

We know that $k_0 = k_s > \bar{k}$ and therefore $k_1 = k_p(k_s)$. For all $m \leq n$ we have then $k_m = k_p^{(m)}(k_s)$ (since by hypothesis $k_p^{(m-1)}(k_s) > \bar{k}$). Since, also by hypothesis, $k_m > k_s$, we have that $k_{m+1} = k_s$ and the result follows.

Proof of Lemma 4

The first four results are obvious from straightforward differentiation. We obtain $\frac{d\tilde{k}'(k)}{dk} \geq 0$ by differentiating equation (15) implicitly:

$$-\frac{\rho'(k')\frac{g_G}{M}}{2M}\frac{d\tilde{k}'_k}{dk} = -\left[\frac{\left(\frac{dR(k,k',0)}{dk'}\frac{d\tilde{k}'(k)}{dk} + \frac{dR(k,k',0)}{dk}\right)\delta_{k,k'} - R(k,k',0)\left(w'(k) + \rho'(k')f_B\frac{d\tilde{k}'(k)}{dk}\right)}{\delta_{k,k'}^2}\right],$$

$$\frac{w'(k)R(k,k',0)}{\delta_{k,k'}^2} - \frac{\frac{dR(k,k',0)}{dk}}{\delta_{k,k'}} = \frac{d\tilde{k}'_k}{dk}\left[-\frac{\rho'(k')\frac{g_G}{M}}{2M} + \frac{\frac{dR(k,k',0)}{dk'}}{\delta_{k,k'}} - \frac{\rho'(k')f_B R(k,k',0)}{\delta_{k,k'}^2}\right].$$

Because $\rho'(k') < 0$, $\frac{dR(k,k',0)}{dk'} > 0$ and $\frac{dR(k,k',0)}{dk} < 0$, the result follows directly.

Proof of Proposition 3

Consider $\{k_t^*\}_{t\geq 0}$ defined inductively as $k_0^* = k_0$, $k_{t+1}^* \in k'(k_t^*)$ according to (20). Let $k_{min}^* := \inf_t k_t^*$ and $k_{max}^* := \sup_t k_t^*$ and define a sequence of beliefs by setting $k'^*_t = k_{t+1}^* \forall t$. Define price sequences by $w_t^* = A(1-\beta)k_t^{*\beta}$ and $\rho_t^* = A\beta {k_{t+1}^*}^{\beta-1}$. Given these prices, define (R_t^*, δ_t^*) as in Proposition 1 .

It remains to check that conditions of Assumption 2 are satisfied for ρ_t^* and w_t^* for all t. Noting that $w(k)$ is increasing in k and $\rho(k')$ is decreasing in k', the hypothesis guarantees exactly this as long as $k_{min}^* \geq k_p(k_{li})$ and $k_{max}^* \leq k_s$. We prove this by showing inductively that the entire sequence $\{k_t^*\}_{t\geq 0}$ lies within $[k_p(k_{li}), k_s]$.

It is true that $k_p(k_{li}) \leq k_0^* \leq k_s$ by hypothesis. Suppose that $k_p(k_{li}) \leq k_t^* \leq k_s$. Then $k_{t+1}^* \geq \min\{k_s, k_p(k_t^*)\} \geq \min\{k_s, k_p(k_{li})\} = k_p(k_{li})$, where the second inequality is from $\frac{dk_p(k)}{dk} > 0$ (Lemma 3) and the equality is due to the hypothesis that $k_p(k_{li}) \leq k_s$. Moreover, $k_{t+1}^* \leq \max\{k_s, k_p(k_t^*)\} \leq \max\{k_s, k_t^*\} \leq k_s$, where the second inequality is due to $\frac{dk_p(k)}{dk} \leq 1$ (Lemma 3 and Assumption 7) and hypothesis $k_p(k_{li}) \leq k_{li}$ and the last inequality is due to the hypothesis.

The existence of cycles is analogous to the proof of Proposition 2. To see that the length of cycles depends on the selection from the beliefs correspondence, consider two sequences $\{k_t^*\}$, and $\{k_t^{**}\}$, with a pessimistic and an optimistic selection of beliefs respectively. Suppose that for some t, k_{t-1}^*,$k_{t-1}^{**} \leq k_{li}$ (the other cases are identical). Then $k_t^* = k_t^{**} = k_s$, and $k_{t+m^*+1}^* = k_s$, where $m^* = \min\{r : k_p^{(r)}(k_s) < k_{li}\}$, and $k_{t+m^{**}+1}^{**} = k_s$, where $m^{**} = \min\{r : k_p^{(r)}(k_s) < k_{hi}\}$. Since $k_{li} < k_{hi}$ and the function $k_p^{(\cdot)}(k_s)$ is decreasing, we have that $m^* \geq m^{**}$. Finally, since $k_p(\cdot)$ is increasing and below the 45 degree line, the cycles in the sequence $\{k_t^{**}\}$ are smaller in amplitude than their counterparts in the sequence $\{k_t^*\}$.

References

Asea, Patrick K. and Brock Blomberg (1998), "Lending cycles," *Journal of Econometrics*, Elsevier, 83(1-2), 89-128.

Azariadis, Costas and Bruce Smith (1998), "Financial Intermediation and Regime Switching in Business Cycles," *American Economic Review*, 88(3), 516-36.

Bernanke, Ben and Mark Gertler (1989), "Agency Costs, Net Worth, and Business Fluctuations," *American Economic Review*, 79(1), 14-31.

Bernanke, Ben, Mark Gertler and Simon Gilchrist (1999), "The financial accelerator in a quantitative business cycle framework," in J. B. Taylor & M. Woodford (ed.), Handbook of Macroeconomics, edition 1, volume 1, chapter 21, 1341-1393.

Berger, Allen N. and Gregory F. Udell (2004), "The institutional memory hypothesis and the procyclicality of bank lending behavior," *Journal of Financial Intermediation,* 13(4), 458-495.

Bester, Helmut (1985), "Screening vs. Rationing in Credit Markets with Imperfect Information" *American Economic Review*, 75(4), 850-855.

Carlstrom, Charles and Timothy Fuerst (1997), "Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis," *American Economic Review*, 87(5), 893-910.

Dell'Ariccia, Giovanni and Robert Marquez (2006), "Lending Booms and Lending Standards," *Journal of Finance*, 61(5), 2511-2546.

De Meza, David and David C. Webb (1987), "Too Much Investment: A Problem of Asymmetric Information," *The Quarterly Journal of Economics,* 102(2), 281-292.

Hellwig, Martin (1987) "Some Recent Developments in the Theory of Competition in Markets with Adverse Selection ," *European Economic Review*, Elsevier, 31(1-2), 319-325.

Greenwald, Bruce C and Joseph E Stiglitz (1993), "Financial Market Imperfections and Business Cycles," *The Quarterly Journal of Economics*, 108(1), 77-114.

Kiyotaki, Nobuhiro and John Moore (1997), "Credit Cycles," *Journal of Political Economy*, 105(2), 211-48.

Lee, Yoonsoo and Toshihiko Mukoyama (2008), "Entry, Exit and Plant-Level Dynamics over the Business Cycle," Working Manuscript, Federal Reserve Bank of Cleveland.

Levy, Amnon and Christopher Hennessy (2007), "Why does capital structure choice vary with macroeconomic conditions?," *Journal of Monetary Economics*, 54(6), 1545-1564.

Lown, Cara and Donald P Morgan (2006), "The Credit Cycle and the Business Cycle: New Findings Using the Loan Officer Opinion Survey," *Journal of Money, Credit and Banking*, 2006, 38(6), 1575-1597.

Martin, Alberto (2006), "Endogenous Credit Cycles," Working Manuscript, Universitat Pompeu Fabra.

Rampini, Adriano (2004), "Entrepreneurial Activity, Risk, and the Business Cycle", *Journal of Monetary Economics*, 51(3), 555,573.

Reichlin, Pietro and Paolo Siconolfi (2004), "Optimal Debt Contracts and Moral Hazard Along the Business Cycle", *Economic Theory*, 24(1), 75-109.

Rothschild, Michael and Joseph E Stiglitz (1976), "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information," *The Quarterly Journal of Economics*, MIT Press, 90(4), pp. 630-49.

Suarez, Javier and Oren Sussman, (1997), "Endogenous Cycles in a Stiglitz-Weiss Economy", *Journal of Economic Theory*, 76, 47-71.

Stiglitz, Joseph E and Andrew Weiss (1981), "Credit Rationing in Markets with Imperfect Information", *American Economic Review*, 71, 393-410.

Williamson, Stephen (1987), "Financial Intermediation, Business Failures, and Real Business Cycles," *Journal of Political Economy,* 95, 1196-1216.

Figure 1.

Charge-off and delinquency rates lag after the cycle.

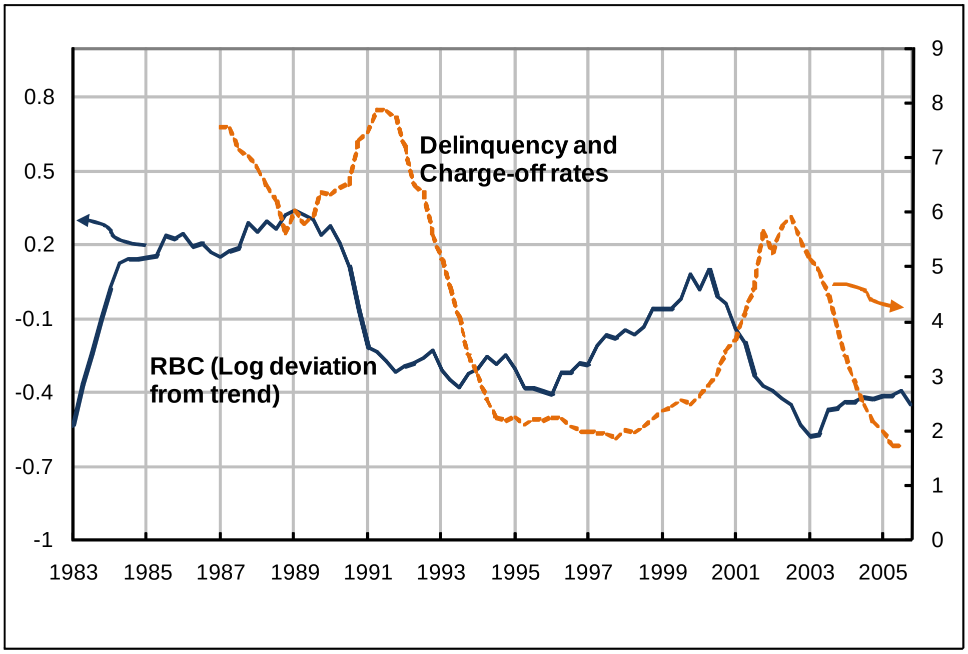


Figure 2.

Equilibrium Contract Determination (for given w and ρ).

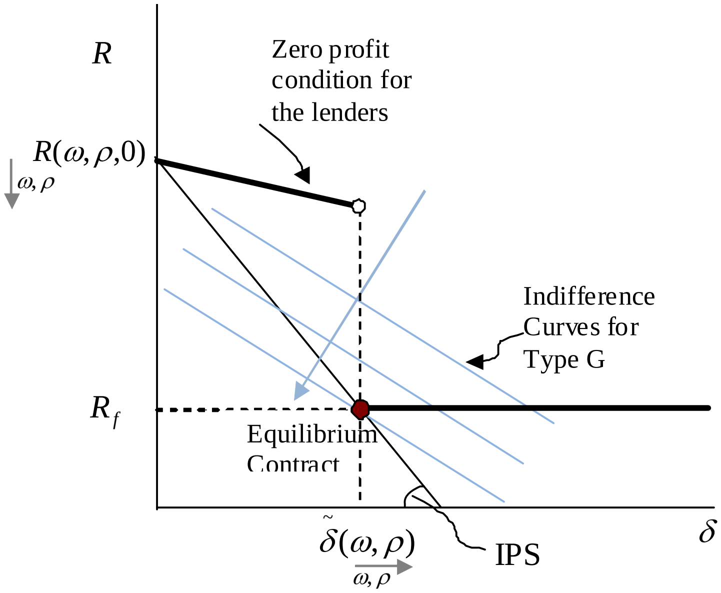


Figure 3.

Simplified Model. Equilibrium Contract Determination, 2 states, $k_H > k_L$.

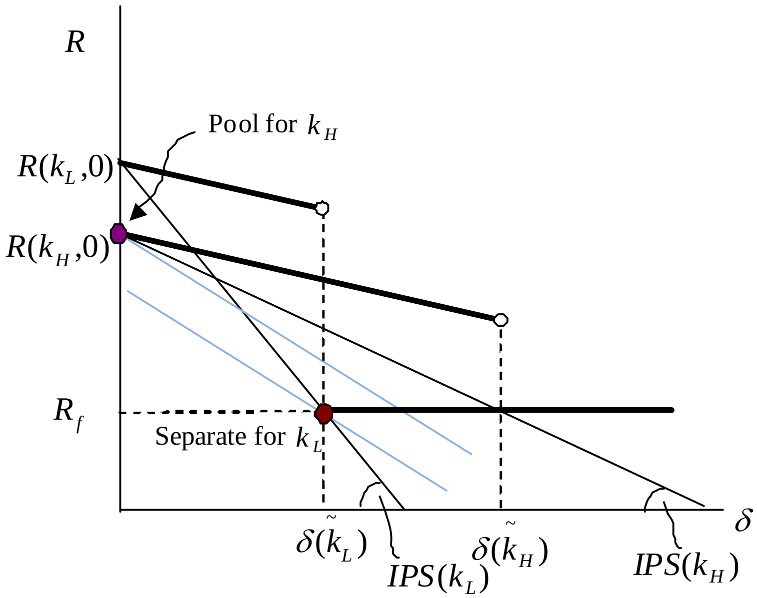


Figure 4.

Transition Function $k_{t+1}(k_t)$ in the Simplified Model.

The case of endogenous cycles: $k_P\left(\bar{k}\right) < \bar{k} < k_S$.

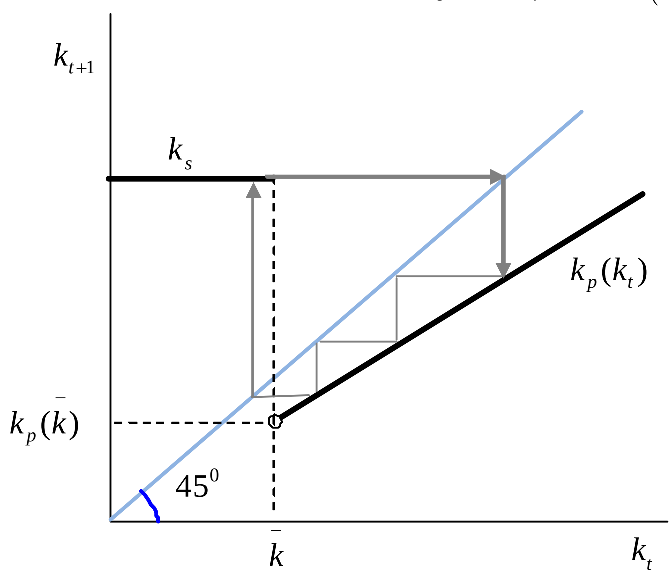


FIGURE 5.

Model with No Externalities. Illustration of $\kappa\left(k,k'\right)$, for a fixed k,

$\tilde{k}(k)$ is defined as a solution to $\frac{1-\rho(k')\frac{g_G}{M}}{2M} = -\frac{R(k,k',0)-R_f}{\tilde{\delta}(k,k')}$.

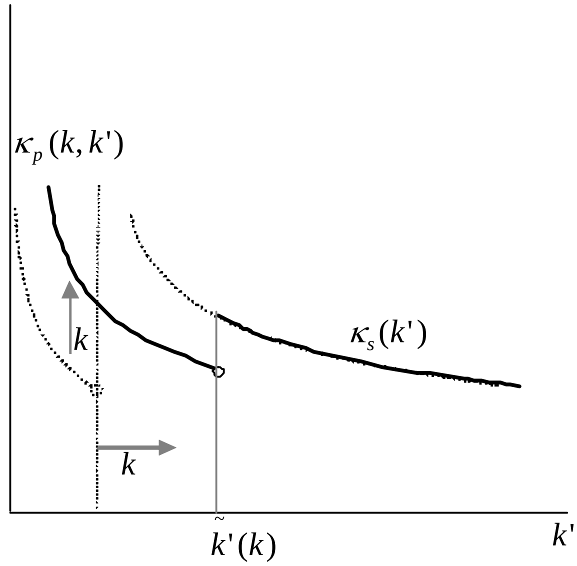


FIGURE 6.

Model with No Externalities. Illustration of k_{hi} and Assumption 5: $\tilde{k}'(k_{hi}) > \kappa_p(k_{hi}, \tilde{k}'(k_{hi}))$.

k_{hi} is the maximum capital that allows for separation, it is given by the solution to $\kappa_s(\tilde{k}'(k)) = \tilde{k}'(k)$.

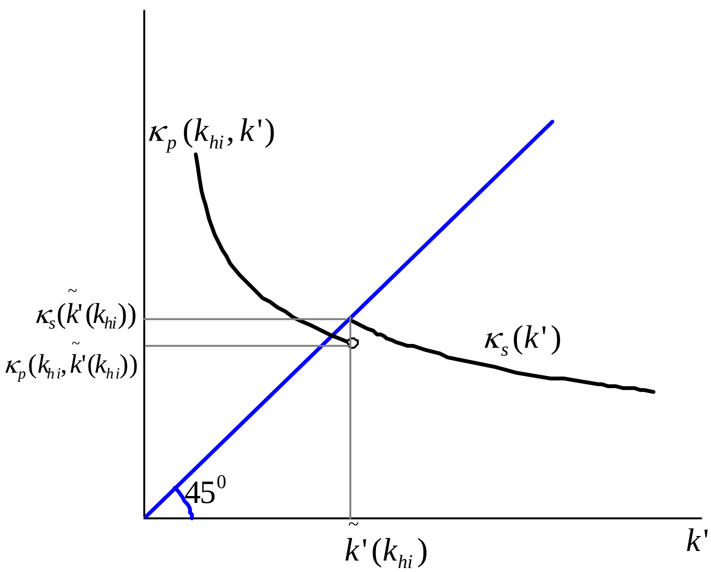


FIGURE 7.

Illustration of k_{li}.

k_{li} is the minimum capital that allows for pooling. If it exists, it solves $\kappa_p(k, \tilde{k}'(k)) = \tilde{k}(k)$.

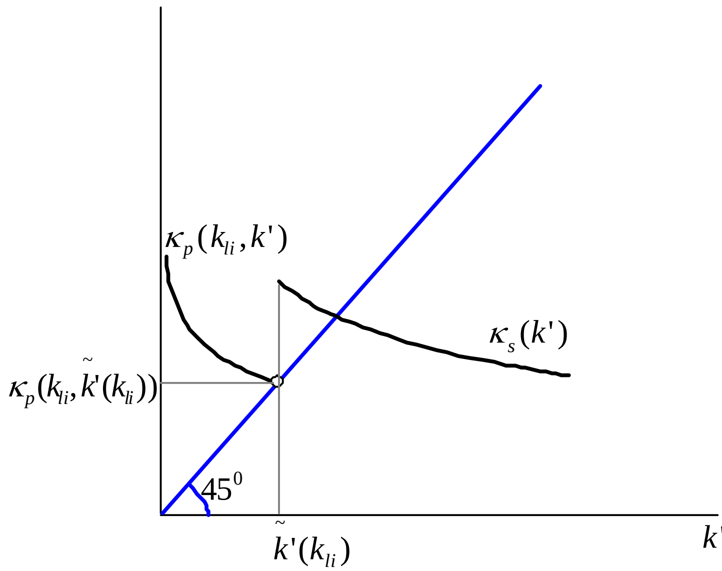


FIGURE 8.

Model with No Externalities. Determination of $k'(k)$, for some capital level $k < k_{li}$.

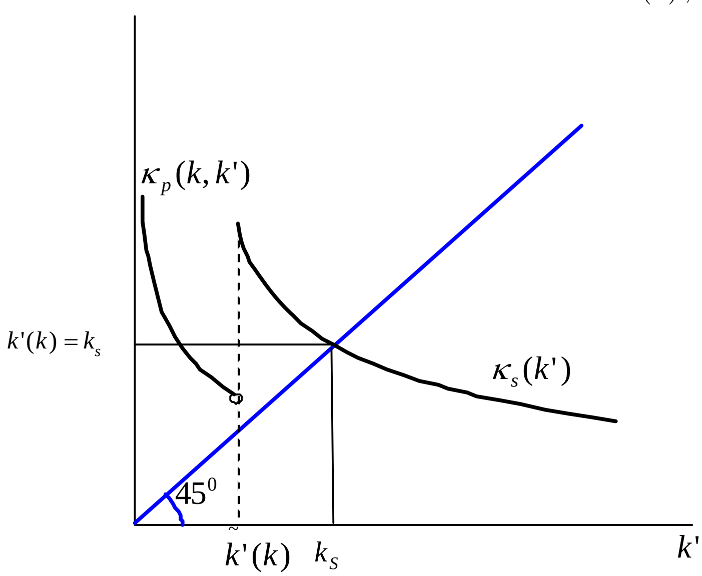


Figure 9.

Model with No Externalities. Determination of $k'(k)$, for some capital level $k \in [k_{li}, k_{hi}]$.

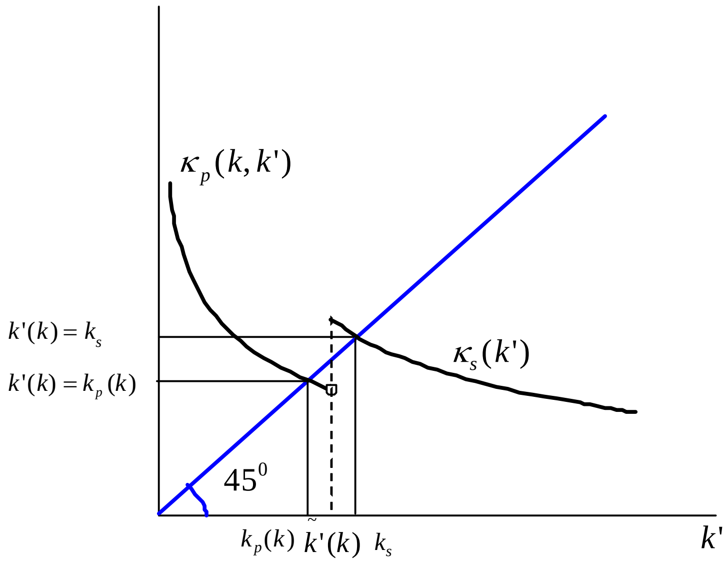


Figure 10.

Model with No Externalities. Determination of $k'(k)$, for some capital level $k > k_{hi}$.

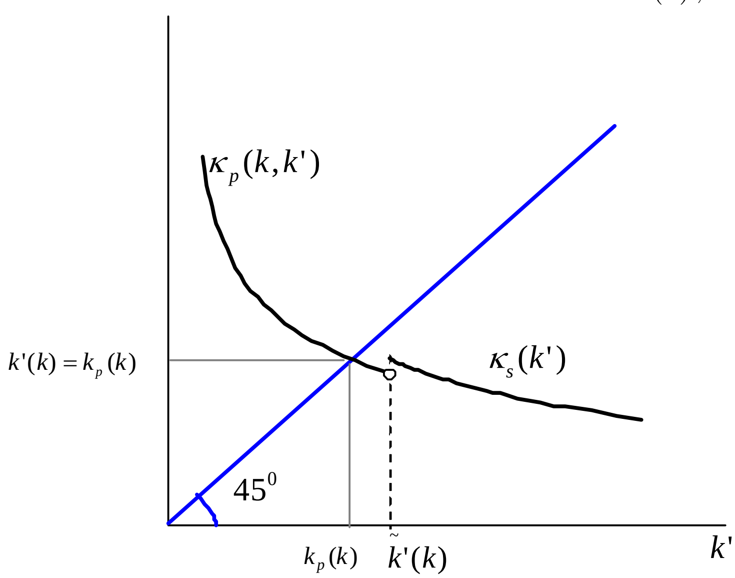


FIGURE 11.

Model with No Externalities. Consistent Belief Correspondence $k'(k)$.

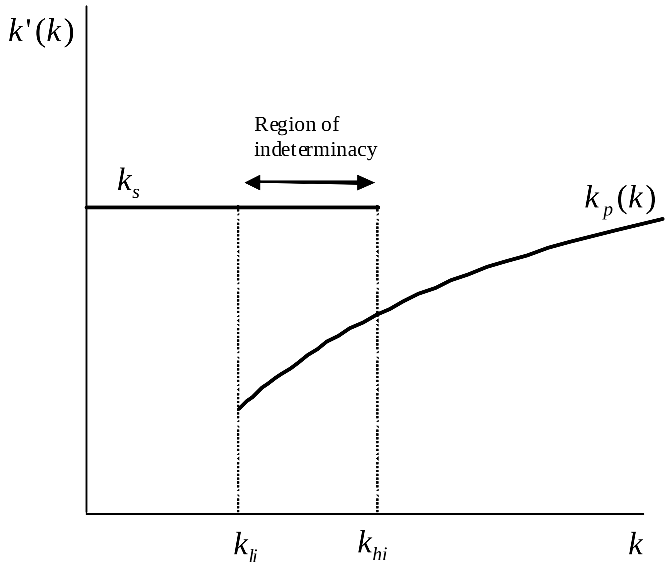


FIGURE 12.

Example 2. Cycles for the Model with no Externalities. The Case of the Pessimistic Belief Selection.

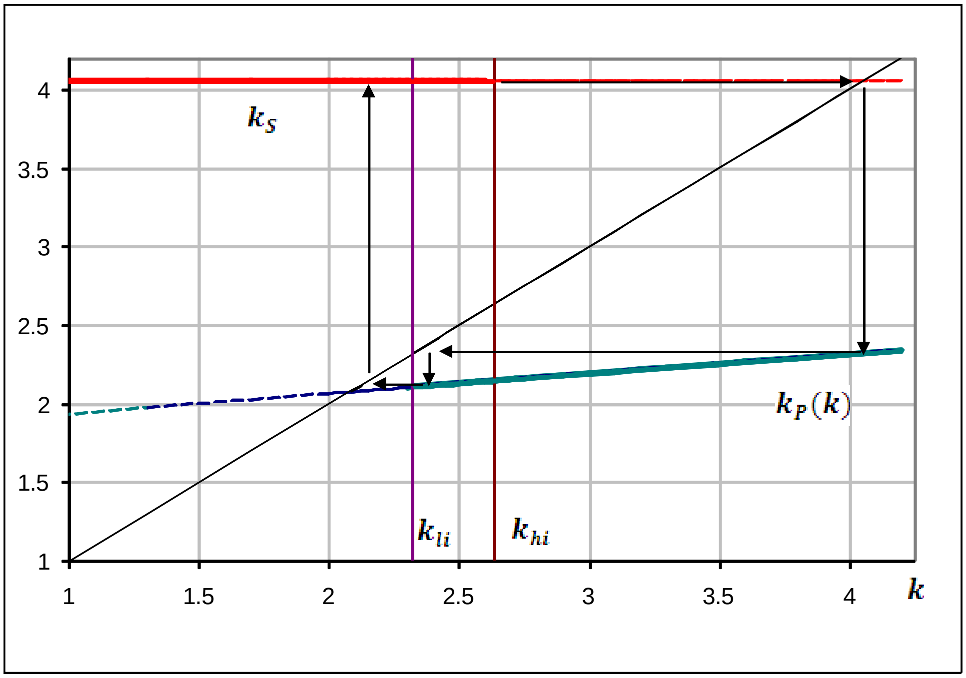
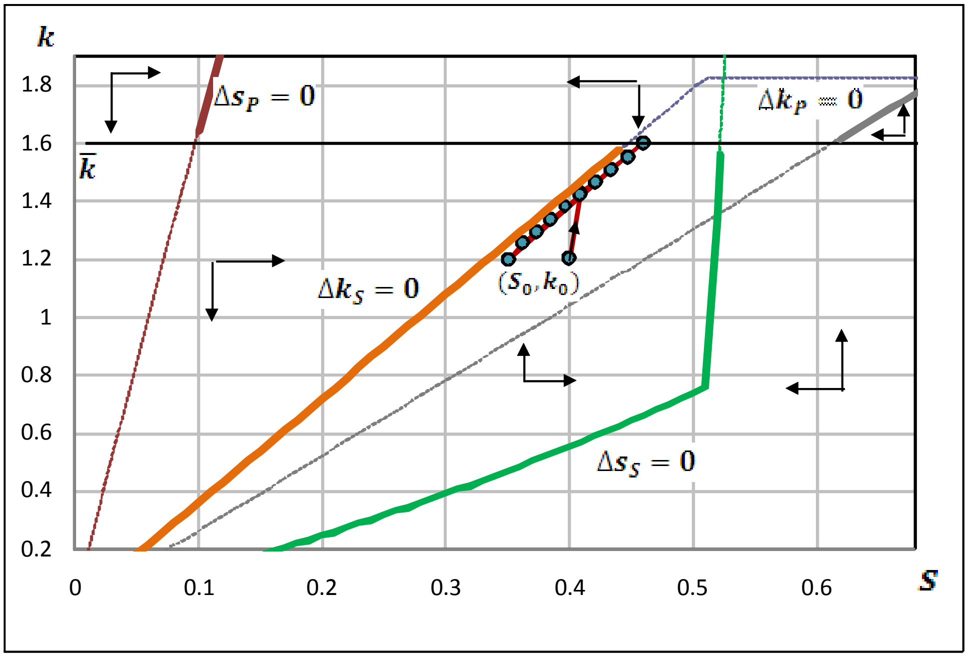


FIGURE 13

Example 3. Cycles for the Model with the Endogenous Supply of Funds (Section VI).

13A. The Phase Diagram.



13B. Capital and the supply of funds dynamics.