Mathematical Programming (2024) 203:1–7
https://doi.org/10.1007/s10107-023-02050-3

PREFACE

Series B

Mathematical Optimization for Fair Social Decisions: A Tribute to Michel Balinski

Mourad Baïou[1] · José Correa[2] · Rida Laraki[3,4]

Published online: 24 January 2024

A conference on mathematical optimization for fair social decisions was organized in tribute of Michel Balinski. This conference was the occasion to overview some of his main contributions in theory and practice.

Michel studied in the United States with graduate degrees from MIT (MSc) and Princeton (PhD). He held professorial positions in mathematics, economics and administrative sciences at Princeton, the University of Pennsylvania, the Graduate School of the City University of New York, Yale and Stony Brook University. Afterwards he moved to France in 1980, he was appointed Research Director "de classe exceptionnelle" of the Centre National de Recherche Scientifique at the École Polytechnique. He was Director of the Laboratoire d'économétrie there until his retirement in 1999. Michel is the founder and first editor of Mathematical Programming and, having been one of the founders of the Mathematical Optimization Society, also served as its president. He is celebrated for, among other things, his work on the diameter of polytopes that arise from the transportation problem and primal-dual algorithms for the matching problem. He developed a new formulation of stable matchings and their generalizations in terms of graphs, leading to the first characterization of the university admissions polytope, and to a generalization of stable matchings to stable allocations (matching opposites in real numbers). In voting theory, he is made foundational contributions to the study of electoral systems and notably his two books on fair representation and Majority Judgment. For his scientific work, Michel was honored with INFORMS John

✉ Mourad Baïou
mourad.baiou@cnrs.fr

José Correa
correa@uchile.cl

Rida Laraki
rida.laraki@gmail.com

1 CNRS, Université Clermont Auvergne Clermont Auvergne INP, Mines Saint-Etienne, LIMOS, 63000 Clermont-Ferrand, France

2 Departamento de Ingenieria Industrial, Universidad de Chile, Santiago, Chile

3 CNRS, LAMSADE, Université Paris-Dauphine PSL, Paris, France

4 Moroccan Center for Game Theory at UM6P, Rabat, Morocco

von Neumann Theory Prize and The Lester Ford Prize of the Mathematical Association of America. This conference was the occasion to overview some of his main contributions in theory and practice.

The conference was jointly organized by The French National Center for Scientific Research (CNRS) and École Polytechnique (l'X) and took place at the CNRS National Headquarters in Paris from December 3rd to 4th, 2019. All the accepted papers in this issue are directly related to Michel's research themes, bridging fundamental science and applied science in the fields of combinatorial optimization, fair representation and apportionment, game theory, and social choice theory. We received many excellent articles and were able to select only 34 articles distributed among 4 groups as follows.

1 Fair Representation and Apportionment

Optimality and Fairness of Partisan Gerrymandering. Lagarde and Tomala [REF] study the problem of optimal partisan gerrymandering, that is, the division of a population of voters into a given number of equally sized districts with the purpose of maximizing the expected number of districts where a given party achieves the majority of votes. After solving the problem and introducing new fairness measures, they show that optimal gerrymandering (full cracking) is the most unfair, and that community districting (full packing) is the most fair.

Phragmén's Voting Methods and Justified Representation. Brill, Freeman, Janson and Lackner [REF] consider three variant of the Phragmén's Voting Methods for selecting committees based on approval ballots. They show that a polynomial-time computable sequential variant satisfies the proportional justified representation axiom (PJR), and that two NP-hard optimization-based variants satisfy in addition to PJR, the perfect representation axiom. Finally, they provide mixed-integer programming algorithms for computing the three variants.

Approval-based apportionment. Brill, Gölz, Peters, Schmidt-Kraepelin and Wilker [REF] consider a model which is in between apportionment and committee elections. Typically, in the problem of apportionment, each voter is asked to approve a single political party. Here, the authors allow the voters to approve as many political parties as they wish and the objective is to allocate a fixed number of seats "proportionally" to the number of votes. They show that (1) the core is nonempty, (2) there exists an aggregation rule that is committee monotonic and that satisfies a proportionality axiom called EJR (Extended Justified Representation), and (3) verifying whether a given committee satisfies EJR/PJR is polynomial-time solvable, while it is coNP complete to verify the core property.

The Maximin Support Method: An Extension of the D'Hondt Method to Approval-Based Multiwinner Elections. Sanchez-Fernandez, Fernández García, Fisteu and Brill [REF] propose a novel approval-based multi-winner voting method that extend the well-known d'Hondt apportionment method which always selects committees that maximize the voter support for the least supported candidate in the committee. They show it to be polynomial-time computable and to satisfy (adjusted versions of) house monotonicity, population monotonicity, and proportional representation.

Apportionment with Parity Constraints. Verdugo and Mathieu [REF] investigate the problem of allocating the seats of a parliament, where parity constraints among candidate types must be met while maintaining party proportionality. This issue recently emerged in the context of the Chilean constitutional convention. The authors provide a formalization of the algorithm employed in the Chilean constitution and introduce a new algorithm based on the concept of biproportionality, as initially introduced by Balinski and Demange. Their analysis encompasses an exploration of the properties of both algorithms and an evaluation of how their solutions compare to the classical fractional solution known as "fair share" (a.k.a matrix scaling).

Note on Axiomatic Properties of Apportionment Methods for Proportional Representation Systems. Palomares, Pukelsheim, and Ramírez [REF] consider the relationships between key axioms in the apportionment literature. The authors study these axioms to evaluate the quality of an apportionment. The main focus is to establish that a subset of these properties are logically independent which complements known results in the literature.

A Simple and Fast Linear-Time Algorithm for Divisor Methods of Apportionment. Reitzig and Wild [REF] introduce a novel algorithm called "SandwichSelect" for determining solutions in divisor methods of apportionment. They demonstrate that this algorithm has a linear complexity on the number of parties in the apportionment model. Furthermore, they present computational results, with a comparative analysis of its performance against some known algorithms.

New Characterizations of Strategy-Proofness under Single-Peakedness. Jennings, Laraki, Puppe and Varloot [REF] present innovative characterizations of strategy-proof voting rules within the context of single-peaked preferences on a line. Notably, they introduce a "grading curve" representation that proves valuable for variable electorates. The work in this paper unifies existing results and provides new characterizations when strategy-proofness is extended to other desirable properties. They also study the computational complexity of the various representations.

Why Webster?. Balinski and Ramírez [REF] review the conditions that make Webster's method especially natural for apportionment problems. It also contains a new result showing that Webster's is the unique parametric method that stays within the quota in a 'local' sense.

2 Matching and Combinatorics

Cutoff stability under distributional constraints with an application to summer internship matching. Aziz, Baychkov and Bíró [REF] introduce a new two-sided matching problem with three different concepts of stability: strong, weak and cutoff stability. What sets it apart from the conventional model is the introduction of a third set of applicants called "supervisors". They first show that determining the existence of a strong stable matching is an NP-hard problem. When relaxing the concept of strong stability, we arrive at weak stability, and finding a stable matching in this context can be achieved in polynomial time. They also provide a polynomial algorithm for finding

a cutoff stable matching, which falls in between strong and weak stability. The paper concludes with the study of the cutoff stable of maximum size problem.

How many matchings cover the nodes of a graph?. Ait-Ferhat, Kiraly, Sebö and Stauffer [REF] study the problem of covering the nodes of the graphs with the smallest possible number of matchings. They present a straightforward polynomial algorithm, which offers a favorable alternative to a previously known algorithm with similar complexity. This algorithm is built upon a structural theorem, which states that a given graph can be covered by matchings if and only if the size of any stable set S is less than times the size of its neighboring set $N(S)$.

Fairness over Time in Dynamic Resource Allocation with an Application in Healthcare. Lodi, Olivier, Pesant and Sankaranarayanan [REF] study the problem of determining the fair allocation of resources to stakeholders, over a time horizon. The first part of this paper is dedicated to presenting theoretical results within specific cases focusing on the structure of the feasible allocations and on the benefit function. In particular, they discuss cases where the fairness is improved when going from the one period case to the multiperiod one. The cases where perfect fairness may be obtained are also discussed. The second part concerns a real world application with extensive computational experiments.

Approximate and Strategyproof Maximin Share Allocation of Chores with Ordinal Preferences. Aziz, Li and Wu [REF] study maximin share fair allocation of m indivisible chores to n agents using only ordinal preferences. The main results concern the approximation guarantees that can be achieved via ordinal algorithms, both with and without strategy-proofness constraints.

Identifying optimal strategies in Kidney Exchange games is $\sum_2^p$-complete. Smeulders, Blom, and Spieksma [REF] aim to understand strategic behavior in kidney exchange programs modeled by a directed graph in which nodes correspond to donor-recipient pairs and arcs correspond to compatibility. They study computational aspects of a two-agent model, representing, for example, a hospital and a national program.

Graphs with G^p—connected medians. Bénéteau, Chalopin, Chepoi and Vaxès [REF] investigate the median problem which is related to the majority judgment. In the median problem, where weights are assigned to its nodes, the objective is to identify a subset of nodes, $M(G)$, that minimizes the total sum of weighted distances between the nodes in $M(G)$ and the remaining nodes. The authors provide a characterization of the graphs G for which $M(G)$ induces a connected subgraph in G^p, which is the graph obtained from G by adding the edges between pair nodes at distance at most p. Additionally, they establish that these graphs may be recognized in polynomial time.

On the minimum $s-t$ cut problem with budget constraints. Aissi and Mahjoub [REF] study the minimum budgeted s-t cut problem. This problem consists in finding the minimum cost $s-t$ cut C, with respect to a specific cost function c^k, such that the cost of with respect to $k-1$ cost functions satisfy $k-1$ budgets, respectively. To address this problem, they introduce a linear relaxation for this problem. By leveraging a partial ordering of its extreme points, they characterize the graphs for which this linear relaxation yields an optimal integral solution. Notably, they also establish the NP-hardness of recognizing these type graphs.

The Computation of Pairwise Stable Networks. Herings and Zhang [REF] dive into network formation problems in which players decide the intensity of each relationship. Recent research has established that a natural stability concept, called pairwise stability can always be achieved. The authors develop a homotopy method to effectively compute pairwise stable networks in general network formation problems.

3 Game Theory

Splitting games over finite sets. Koessler, Laclau, Renault and Tomala [REF] prove that the splitting game over finite sets and terminal payoffs has a value, that can be characterized thanks to the famous Mertens and Zamir operator (MZ) and define an operator on matrices that leads to an approximation of the (MZ) operator.

The Price of Anarchy in Series–Parallel Network Congestion Games. Hao and Michini [REF] address an open problem in the characterization of the price of anarchy of symmetric network congestion games with affine latency functions. In this context, the authors obtain improved bounds for the case of series–parallel graphs

The Price of Anarchy in Routing Games as a Function of the Demand. Cominetti, Dose, and Scarsini [REF] study the price of anarchy in single-source single-sink nonatomic congestion games as a function of the demand. The authors study the continuity, differentiability, and monotonicity of Wardrop equilibria in the case of general nondecreasing cost functions. They also show explicit results for the evolution of the Price of Anarchy for affine cost functions.

A unified stochastic approximation framework for learning in games. Mertikopoulos, Hsieh and Cevher [REF] develop a unified stochastic approximation framework for analyzing online learning algorithms in game theory based on a flexible primal-dual Robbins-Monro template. The proposed framework allows to obtain a broad range of new convergence results, both asymptotic and in finite time, in both continuous and finite games.

Strong Substitutes: Structural Properties, and a New Algorithm for Competitive Equilibrium Prices. Baldwin, Bichler, Fichtl, Klemperer [REF] study structural properties and algorithms for strong substitutes product-mix auctions, a well-studied class of combinatorial auctions. The authors give a new linear programming-based algorithm for computing equilibrium prices.

No-regret algorithms in on-line learning, games and convex optimization. Sorin [REF] highlights the links between no-regret algorithms used in online learning, game theory and convex optimization. It offers a unifying point of view by extracting some basic principles and underlying the analogy between continuous and discrete time analysis. As a main result, it is proved that the three algorithms: projected gradient, mirror descent and dual averaging satisfy the no-regret property, with different performances.

The Core of a Transferable Utility Game as the Solution to a Public Good Market Demand Problem. Edelman, Van der Linden, Weymark [REF] provide three characterizations of the core of a cooperative game with transferable utility (TU): as a set of

prices that arise in an economy where the coalitions are public goods; as a subdifferential of the cover function evaluated at the grand coalition; and as the intersection of the subdifferentials of the cover functions of person-specific TU games.

Utility/Privacy Trade-off as Regularized Optimal Transport. Boursier and Perchet [REF] propose to model the trade-off between utility loss and maintaining the privacy of strategic information transmission via a Sinkhorn entropy-transport minimisation problem. They propose an extensive list of minimization schemes and numerical experiments and apply their model to design an online learning algorithm in a repeated auction.

A Finite Characterization of Perfect Equilibria. Callejas, Govindan and Pahl [REF] study perfect equilibria, which are a refinement of Nash equilibria. Govindan and Klumpp showed in 2002 that a profile is a perfect equilibrium of a finie game if and only if it is a solution of a finite number of equalities and inequalities involving polynomials. This paper gives an explicit upper bound on the size of this system using the Nash curve selection lemma and the Lojasiewicz inequality.

Absorption Paths and Equilibria in Quitting Games. Ashkenazi-Golan, Krasikov, Rainer and Solan [REF] define, in the context of quitting games, the concept of sequentially 0-perfect absorption paths, which are shown to be limits of ε-equilibrium strategy profiles as ε goes to 0. They prove that all quitting games that do not have simple equilibria have a sequentially 0-perfect absorption path, and prove the existence of sequentially 0-perfect absorption paths in a new class of quitting games.

Stackelberg Pricing Games with Congestion Effects. Harks and Schedel [REF] study a Stackelberg pricing game with multiple leaders and a continuum of followers. They characterize the best responses of the players as the optimal solutions of two ordinary optimization problems and use this characterization to establish the existence of equilibria in pure strategies.

Characterization of TU games with stable cores by nested balancedness. Grabisch and Sudhölter [REF] prove that a balancedness condition is necessary and sufficient for core stability in cooperative games with transferable utility (TU). They also show that the set of pre-imputations that are maximal with respect to outvoting is equal to the core if and only if the core is stable, and that a TU game has a super-stable core if and only if the game is vital extendable.

4 Social Choice Theory

On the resolution of cross-liabilities. Demange [REF] contributes to the study of a generalization of the classic bankruptcy model of O'Neill (1982), by allowing mutual liability, the goal being to derive a form of proportional solution to the problem. She does so in two different ways: through the maximization of an entropy index and by following an axiomatic approach, very much in the spirit of Balinski and Demange (1989).

Consistent Queueing Rules. Thomson and Velez [REF] study a queueing problem where a group of agents are waiting for a service and incur a cost of waiting that is

proportional to the time they wait. They characterize the subsolutions of the no-envy solution that are anonymous, consistent, conversely consistent, continuous and budget monotone. In particular, they show that there are infinitely many proper consistent subsolutions, and provide a characterization of minimal consistent subsolutions satisfying these axioms.

Truthful ownership transfer with expert advice. Caragiannis, Filos-Ratsikas, Nath, and Voudouris [REF] introduce a framework, combining mechanism design with and without money, when a single good is to be sold to one of two bidders. Additionally, an expert provides advice and obtains utility depending on the allocation of the good. The authors obtain nearly tight results for the social welfare obtained via truthful mechanisms.

Truthful Facility Assignment with Resource Augmentation: An Exact Analysis of Serial Dictatorship. Caragiannis, Filos-Ratsikas, Frederiksen, Arnsfelt Hansen, and Tan [REF] design mechanisms for a facility assignment problem in which agents with private locations are to be assigned to publicly known facilities. The authors study the performance, in term of social welfare, of the truthful serial dictatorship mechanism with facilities of increased capacity.

Fair division of graphs and of tangled cakes. Igarashi and Zwicker [REF] extend the classic cake-cutting model, from simple cakes taking the shape of an interval or a pie, to cakes that are general graphs or "tangles". The authors' main result is to determine the tangles that guarantee envy-free allocations for any number of agents.