Comparative statics in strategic form games*

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Abstract

This paper presents new comparative statics results for strategic form games, expanding the range of games where lattice theoretic tools can be applied. We introduce a dominance condition that ensures payoff shocks result in increased Nash equilibria, even in games without strategic complementarities. Our results are straigtforward to apply. We derive new comparative statics results for contests and Cournot games.

1 Introduction

Understanding how exogenous variables impact equilibrium behavior is fundamental in economics. For instance, in a rent-seeking context, changes in prizes can significantly impact rent-seeking behavior. Two main approaches are used to obtain comparative statics results in games. One approach analyzes the impact of changes in the economic environment using the implicit function theorem (Nti, 1997). This method works in a differentiable environment and, more critically, only when the exogenous shocks are small. The second approach restricts attention to games with strategic complements and applies lattice theory (Milgrom and Roberts, 1990; Vives, 1990). This approach allows for global comparative statics results but excludes important games, such as contests and Cournot games, that do not exhibit strategic complementarities. Consequently, deriving general insights for these games becomes challenging.

We present new comparative statics results for strategic form games. Our main results provide conditions under which the set of Nash equilibria increases when payoffs change, even in situations where payoffs are neither complements nor substitutes. A corollary to our main theorem extends the monotone comparative statics results for games with strategic complements (Milgrom and Roberts, 1990; Vives, 1990, 1999) to games that fail to exhibit any strategic complementarity.

In strategic form games, payoff shocks have both direct and strategic effects. In games that lack strategic complements, strategic effects obscure the analysis because their impact on strategies may oppose the direct effect of the payoff shock. To address these difficulties, we introduce a new dominance condition that governs how exogenous payoff shocks affect the game. The dominance condition is satisfied if, for each player, her incremental utility from increasing her action is higher with the new payoffs than with the original payoffs uniformly over a subset of rivals' actions. Intuitively, our dominance condition ensures that the payoff shock

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is strong enough to motivate each player to increase her actions regardless of how other players react after the shock.

Our results are easy to use in applications. In games with differentiable payoffs, it is sufficient to check an inequality involving derivatives of payoffs before and after the payoff shock. We illustrate our approach by deriving new comparative statics results for contests and Cournot games.

A conceptual innovation in this paper is the introduction of the monotone envelopes for the best response maps. The upper (lower) monotone envelope is the smallest (largest) non-decreasing function that is above (below) the best response map. The monotone envelopes are a useful construction to compare Nash equilibria of the games before and after the exogenous payoff shock. Our construction could be useful in other applications of fixed-point theorems in economics.

We contribute to the literature on comparative statics in strategic form games by expanding the set of games where lattice theoretic tools can be employed (Topkis, 1979; Milgrom and Roberts, 1990; Vives, 1990, 1999; Zhou, 1994). In games that lack strategic complements, unambiguous comparative static results are hard to obtain. Thus, the literature derives results by restricting attention to aggregative games (Acemoglu and Jensen, 2013) or by imposing symmetry restrictions Gama and Rietzke (2019). Other results require the analyst either to compute the best responses (Roy and Sabarwal, 2010) or to build a strategic form game that can be used to pivot the comparative statics (Amir and Rietzke, 2023). Our results apply to a general class of games under restrictions on payoffs that are simple to check in examples

2 Model

2.1 Strategic form games

We consider strategic form games with a finite set of players I, in which each player has a nonempty set of actions A_i, with payoff functions $u_i \colon \prod_i A_i \to \mathbb{R}$. We restrict attention to strategic environments in which A_i is a complete lattice, given some partial order $\geq_i$, for each i.[1] Best responses are non-empty, that is, for all i and all a_{-i}

$$|\arg\max_{a_i \in A_i} u_i(a)| \neq \emptyset. \tag{2.1}$$

For each i, we assume u_i is supermodular in a_i, that is, for all $a_{-i} \in A_{-i}$ and all $a_i, a_i' \in A_i$,

$$u_i(\max\{a_i, a_i'\}, a_{-i}) + u_i(\min\{a_i, a_i'\}, a_{-i}) \geq u_i(a_i, a_{-i}) + u_i(a_i', a_{-i}) \tag{2.2}$$

Payoff u_i is supermodular when A_i is an interval in the real line. All games considered in this paper have non-empty best responses and are supermodular (thus satisfy conditions (2.1) and (2.2)).

Our main goal is to provide comparative statics results for the set of Nash equilibria. We will fix the set of players, the set of actions, and identify the game with its payoff functions $u = (u_i)_{i \in I}$. Let $\mathrm{NE}(u) \subseteq \prod_i A_i$ be the set of Nash equilibria for game u.

We say that u is *regular* if for all i, $u_i(a)$ is continuous in $a \in A$, A_i is convex, and $u_i(a)$ is quasi-concave

[1] We will omit the dependence of the partial order on i, as it will be clear from context. When comparing vectors, we consider the product order.

in a_i. We say that u is a game with *strategic complements* if for all i, all $x_{-i} \le a_{-i}$,

$$a_i \in A_i \mapsto u_i(a_i, a_{-i}) - u_i(a_i, x_{-i})$$

is non-decreasing in a_i. When $a_i \in A_i \mapsto u_i(a_i, a_{-i}) - u_i(a_i, x_{-i})$ is non-increasing in a_i for all $x_{-i} \le a_{-i}$, we say that the game is of *strategic substitutes*.

2.2 Comparing games

Consider two games characterized by payoffs $U = (U_i)_{i \in I}$ and $u = (u_i)_{i \in I}$. We say that U *dominates* u *above* $\underline{a} \in A$ if for all i, all $a_{-i}, x_{-i} \in A_{-i}$, with $\underline{a}_{-i} \le x_{-i} \le a_{-i}$, and all $a'_i, a_i \in A_i$ with $a'_i \ge a_i$ and $a'_i \ne a_i$,

$$u_i(a'_i, x_{-i}) \ge u_i(a_i, x_{-i}) \implies U_i(a'_i, a_{-i}) > U_i(a). \tag{2.3}$$

We say that U *dominates* u *below* $\bar{a} \in A$ if for all i, all $a_{-i}, x_{-i} \in A_{-i}$, with $\bar{a}_{-i} \ge x_{-i} \ge a_{-i}$, and all $a'_i, a_i \in A_i$ with $a'_i \ge a_i$ and $a'_i \ne a_i$,

$$u_i(a'_i, a_{-i}) \ge u_i(a_i, a_{-i}) \implies U_i(a'_i, x_{-i}) > U_i(a_i, x_{-i}). \tag{2.4}$$

We say that U *dominates* u if U dominates u above $\min A$ or, equivalently, below $\max A$.

The restriction to payoffs such that U dominates u is a strong version of the single crossing condition usually employed to derive comparative statics results for optimization problems (Milgrom and Shannon, 1994). The single crossing condition is also imposed to derive comparative statics results for games with strategic complements (Milgrom and Roberts, 1990). Since we are dealing with games that need not be of strategic complements, we need to strengthen the single crossing condition. The following proposition shows simple sufficient conditions ensuring that U dominates u.

Proposition 1. *a. Suppose that for all i, all $a_{-i}, x_{-i} \in A_{-i}$, with $x_{-i} \le a_{-i}$, and all $a'_i, a_i \in A_i$ with $a'_i \ge a_i$ and $a'_i \ne a_i$,*

$$u_i(a'_i, x_{-i}) - u_i(a_i, x_{-i}) < U_i(a'_i, a_{-i}) - U_i(a).$$

Then, U dominates u.

b. Suppose that for all i, A_i is a closed interval, and u_i and U_i are differentiable in a_i. Suppose that for all i, all $a_{-i}, x_{-i} \in A_{-i}$, with $x_{-i} \le a_{-i}$,

$$\frac{\partial u_i(a_i, x_{-i})}{\partial a_i} < \frac{\partial U_i(a_i, a_{-i})}{\partial a_i}$$

Then, U dominates u.

3 Main result

We now state our main result.

Theorem 1. *a. Assume that U is either of strategic complements or regular. Let $\underline{a} \in \mathrm{NE}(u)$ and assume that U dominates u above $\underline{a}$. Then, there exists $\bar{a} \in \mathrm{NE}(U)$ with $\bar{a} \ge \underline{a}$.*

b. Assume that u is either of strategic complements or regular. Let $\bar{a} \in \mathrm{NE}(U)$ and assume that U dominates u below $\bar{a}$. Then, there exists $\underline{a} \in \mathrm{NE}(u)$ with $\bar{a} \geq \underline{a}$.

Theorem 1 provides conditions under which one can compare equilibria of games U and u. In particular, under some conditions, for each equilibrium of game u (resp. game U) we can find a larger (resp. smaller) equilibrium for game U (resp. game u). The theorem may be applied even if both u and U fail to be of strategic complements, provided the dominance condition holds. Checking whether U dominates u is simple (see Proposition 1) and, as we will show, the theorem can be easily applied to specific examples.

Since U and u need not be of strategic complements, their sets of Nash equilibria need not be ordered (Monaco and Sabarwal, 2016). In our general framework, it is not possible to ensure that there exists an element $\bar{a} \in \mathrm{NE}(U)$ greater than or equal to all Nash equilibria $\underline{a} \in \mathrm{NE}(u)$. The following result covers the case in which both U and u are regular.

Corollary 1. *Let U and u be regular, with U that dominates u.*

a. $\mathrm{NE}(U)$ is larger than $\mathrm{NE}(u)$ in the weak set order: for each $\underline{a} \in \mathrm{NE}(u)$ there exists $\bar{a} \in \mathrm{NE}(U)$ such that $\bar{a} \geq \underline{a}$, and for each $\bar{a} \in \mathrm{NE}(U)$ there exists $\underline{a} \in \mathrm{NE}(u)$ such that $a' \geq a$.

b. Assume that either u or U is a game with strategic complements.[2] Then, $NE(U)$ is larger than $\mathrm{NE}(u)$ in the extremal set order: there exist Nash equilibria $a^* \in \mathrm{NE}(U)$ and $a_* \in \mathrm{NE}(u)$ such that for all $\bar{a} \in \mathrm{NE}(U)$ and all $\underline{a} \in \mathrm{NE}(u)$

$$a_* \leq \bar{a} \quad a^* \geq \underline{a}.$$

The following corollary is the well-known comparative statics result for games with strategic complements (Milgrom and Roberts, 1990).

Corollary 2. *Let U and u be games with strategic complements such that for all i, all $a_{-i} \in A_{-i}$, and all $a'_i, a_i \in A_i$ with $a'_i \geq a_i$ and $a'_i \neq a_i$,*

$$u_i(a'_i, a_{-i}) \geq u_i(a) \implies U_i(a'_i, a_{-i}) > U_i(a). \tag{3.1}$$

Then $\mathrm{NE}(U)$ is larger than $\mathrm{NE}(u)$ in the weak set order.

The next corollary applies to games with strategic substitutes.

Corollary 3. *Let U be regular and $\underline{a} \in \mathrm{NE}(u)$. Assume that u is a game with strategic substitutes and that for all i and all $a_{-i} \geq \underline{a}_{-i}$,*

$$U_i(a) - u_i(a_i, \underline{a}_{-i}) \text{ is non-decreasing in } a_i. \tag{3.2}$$

Then, there exists $\bar{a} \in \mathrm{NE}(U)$ with $\bar{a} \geq \underline{a}$.

In games with strategic substitutes, a change in payoffs that increases best responses may not be enough to ensure the equilibrium set increases. Indeed, an increase in rivals' actions pushes a player to reduce her strategic response. See Figure 1a. Condition (3.2) ensures that best responses move to the top right so that the equilibrium set increases. See Figure 1b.

[2] In this case, either $\mathrm{NE}(u)$ or $\mathrm{NE}(U)$ has a smallest and a largest element.

(a) Game with strategic substitutes. Best responses in thick blue are decreasing. After payoffs change, both best responses increase to the dashed red functions. However, the equilibrium does not increase.

(b) Best responses in thick blue are decreasing. After payoffs change, both best responses increase to dashed red functions. Under condition (3.2), both best responses move to the top right so that the equilibrium increases.

3.1 Proof of Theorem 1

We now discuss the proof of the first part of Theorem 1. Let $br_i(a_{-i})$ (resp. $BR_i(a_{-i})$) be the best response correspondence for player i in game u (resp. game U). Since A_i is a complete lattice, we define

$$\bar{b}r_i(a_{-i}) = \sup\{br_i(a_{-i})\} \quad \text{and} \quad \underline{b}r_i(a_{-i}) = \inf\{br_i(a_{-i})\}$$

We define $\bar{B}R_i$ and $\underline{B}R_i$ analogously. We consider the *monotone upper envelope* of br_i:

$$br_i^+(a_{-i} \mid \underline{a}) = \sup\left\{\bar{b}r_i(x_{-i}) \mid \underline{a}_{-i} \leq_{-i} x_{-i} \leq_{-i} a_{-i}\right\}.$$

The monotone upper envelope br_i^+ is well defined for all $a_{-i} \geq \underline{a}_{-i}$. The monotone upper envelope $br_i^+(a_{-i} \mid \underline{a})$ is a non-decreasing function that is greater than or equal to $br_i(a_{-i})$, but coincides with $br_i(a_{-i})$ whenever br_i is non-decreasing over $\underline{a}_{-i} \leq x_{-i} \leq a_{-i}$. See Figure 2. The introduction of the monotone envelope is a key conceptual step to establish Theorem 1.[3]

The proof of Theorem 1 follows from two observations. The first observation is that we can compare solutions to fixed point equations, even when the operators are not monotone.[4]

Lemma 1. *Assume one of the following conditions hold:*

i. For all i, A_i is convex, BR_i is upper semi-continuous and convex-valued; or

ii For all i, $\bar{B}R_i(a_{-i})$ is non-decreasing in a_{-i}.

Let $\underline{a} \in br(\underline{a})$. Assume that for all i and all a_{-i}, $\underline{B}R_i(a_{-i}) \geq br_i^+(a_{-i} \mid \underline{a})$. Then, there exists $\bar{a} \geq \underline{a}$ such that $\bar{a} \in BR(\bar{a})$.

This lemma ensures that for any fixed point $\underline{a}$ of br there exists another fixed point $\bar{a} \geq \underline{a}$ of BR.

[3]To prove the second part of Theorem 1, we introduce the *monotone lower envelope*

$$BR_i^-(a_{-i} \mid \bar{a}) = \inf\left\{BR_i(x_{-i}) \mid \bar{a}_{-i} \geq x_{-i} \geq a_{-i}\right\}.$$

The monotone lower envelope $BR_i^-(a_{-i})$ is a non-decreasing function that is less than or equal to $BR_i(a_{-i})$.

[4]Lemma 1 holds even when BR and br are not obtained as best responses in normal form games. As a result, Lemma 1 can be applied to other problems in economics, such as matching problems.

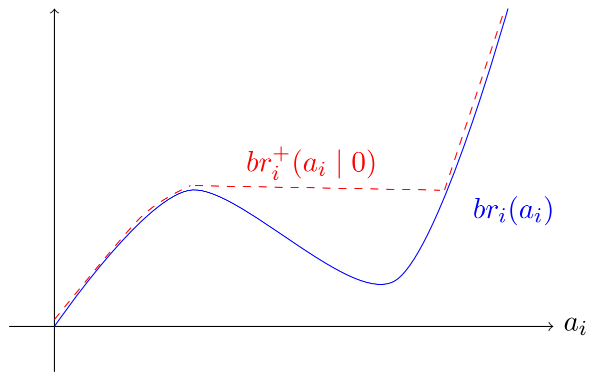

Figure 2: Monotone upper envelope for $\underline{a} = 0$.

Conditions (i) or (ii) immediately ensure the fixed point $\bar{a} \in BR(\bar{a})$ exists. The Lemma shows that when $\underline{B}R_i(a_{-i}) \geq br_i^+(a_{-i} \mid \underline{a})$ for all a_{-i}, the fixed point $\bar{a}$ can be taken so that $\bar{a} \geq \underline{a}$.

Lemma 2. *Under the conditions of Theorem 1 part a, for all i and all $a_{-i} \in A_{-i}$, $\underline{B}R_i(a_{-i}) \geq br_i^+(a_{-i} \mid \underline{a})$.*

Lemma 2 ensures that BR dominates the upper envelope $br^+(\cdot \mid \underline{a})$ when (2.4) holds. Theorem 1 follows by noticing that either A_i is convex and BR_i is upper semicontinuous and convex-valued, or $\bar{B}R_i$ is non-decreasing. As a result, Lemma 2 and Lemma 1 yield the proof of Theorem 1.

4 Applications

4.1 Contests

Consider a contest game in which the action space of each agent is a closed interval $A_i \subseteq \mathbb{R}$ and the payoff function is given by

$$u_i(a) = V_i p_i(a) - c_i(a_i)$$

where $p_i(a) \geq 0$ is the probability with which player i obtains a prize. Players have heterogeneous valuation for the prize so that agent i values the prize that in $V_i > 0$. The effort cost each player incurs equals $c_i(a_i)$. We assume that $\sum_{i \in I} p_i(a) \leq 1$ so that the prize need not be awarded. We assume that $p_i(a)$ and $-c_i(a_i)$ are continuously differentiable, increasing and concave in a_i.

Contest games are important in economics. They can be used to understand rent-seeking behavior and patent races (Tullock, 1980; Dixit, 1987; Loury, 1979). However, contest games are hard to analyze. In most formulations, contest games exhibit non-monotonic best responses. Consequently, comparative statics results for contest games impose important parametric restrictions or symmetry (Nti, 1997; Acemoglu and Jensen, 2013; Jensen, 2016). The very simple question of whether increasing the prize motivates more efforts can be answered only in special environments.

We can use our results to shed light on the comparative statics of contest games. Suppose that the prize increases so that each player i now values the prize in $\bar{V}_i$ (with $\bar{V}_i > V_i$). The contest game with prize valuations $\bar{V}$ will dominate the original game provided

$$\bar{V}_i \frac{\partial p_i(a)}{\partial a_i} > V_i \max_{x_{-i} \leq a_{-i}} \frac{\partial p_i(a_i, x_{-i})}{\partial a_i} \quad \text{for all } a \in A \tag{4.1}$$

This condition says that in order to ensure dominance, the prize difference must be sufficiently significant. Assuming that $\inf_{i,a\in A} \frac{\partial p_i(a)}{\partial a_i} > 0$ and noting that our contest games are regular, Corollary 1 implies that a sufficiently significant increase in the prize increases (in the weak set order) the set of Nash equilibria.[5]

Contest games have non-monotonic best responses and, as a result, a marginal increase in the prize has ambiguous impact on the equilibrium set. Condition (4.1) says that even when the game is not of strategic complements, a significant increase in the prize will always motivate the players to increase their equilibrium actions.

4.2 Cournot oligopoly

Consider a Cournot game in which each firm i decides a quantity $q_i \in [0,1]$. The inverse demand function is given by the strictly decreasing function $P(Q)$, where $Q = \sum_i q_i$. Costs are linear and heterogeneous so that the cost of firm i is $c_i q_i$, with $c_i \geq 0$. We assume that P is differentiable and concave so a Nash equilibrium is guaranteed to exist.

Suppose now that a technological change reduces costs to $\tilde{c}_i < c_i$ for all i. Do firms produce more after the technological change? The reduction in marginal costs increases best responses, but the strategic effects arising in games with strategic substitutes obscure the analysis. Our framework can be used to describe the impact of the technological change on equilibrium outcomes. Let $\underline{q} \in \mathrm{NE}(c)$ (here we identify the game with its costs). In our Cournot framework, Condition (3.2) reduces to

$$\max_{q_i\in[0,1],q_{-i}\geq \underline{q}_{-i}} \frac{\partial}{\partial q_i}\Big\{P(q_i + \sum_{j\neq i} \underline{q}_j)q_i - P(Q)q_i\Big\} < c_i - \tilde{c}_i \tag{4.2}$$

for all i and all profile q. This condition says that the reduction in all marginal costs must be significant enough. Under (4.2), there exists $\bar{q} \in \mathrm{NE}(\tilde{c})$ such that $\bar{q} > \underline{q}$.

These comparative statics result complements several recent studies. Proposition 4 in Acemoglu and Jensen (2013) implies that when the marginal cost of one of the firms decrease, then that firm produces more and the other firms produce less. Our analysis is different as we allow the costs of all firms to decrease. More recently, Gama and Rietzke (2019) derive comparative statics results for symmetric Cournot games. Our results apply to Cournot games that need not be symmetric, but cost shocks need to be significant.

Proofs

Proof of Proposition 1. It is an immediate extension of the observation that the single crossing condition holds under increasing differences; see Vives (1999).

□

Proof of Lemma 1. Restrict the monotone upper envelope map br^+ to the set $\prod_i[\underline{a}_i, \max A_i]$ and note that for any i and any $a_{-i} \in \prod_{j\neq i}[\underline{a}_j, \max A_j]$,

$$br_i^+(a_{-i} \mid \underline{a}) \geq br_i^+(\underline{a}_{-i} \mid \underline{a}) \geq \bar{b}r_i(\underline{a}_{-i}) \geq \underline{a}_i$$

[5] In most formulations of contest games, $p_i(a) = \frac{h_i(a_i)}{R+\sum_j h_j(a_j)}$, with $h_i(a_i)$ concave and having strictly positive derivative, and $R > 0$. In those cases, $\inf_{i,a\in A} \frac{\partial p_i(a)}{\partial a_i} \geq \frac{h_i'(\max A_i)}{R+h_i(\max A_i)} > 0$.

where the first inequality follows since br_i^+ is monotone and the second inequality follows since br_i^+ is above br_i. It follows that

$$br^+(\prod_i [\underline{a}_i, \max A_i]) \subseteq \prod_i [\underline{a}_i, \max A_i]$$

By Tarski fixed point theorem, there exists $a^+ \in \prod_i [\underline{a}_i, \max A_i]$ such that $a^+ = br^+(a+) \geq \underline{a}$.

Consider now the correspondence BR restricted to $\prod_i [a_i^+, \max A_i]$. Take $a_{-i} \in \prod_{j \neq i} [a_j^+, \max A_j]$ and note that

$$\inf\{BR_i(a_{-i})\} \geq br_i^+(a_{-i} \mid \underline{a}) \geq br_i^+(a_{-i}^+ \mid \underline{a}) = a_i^+.$$

The first inequality follows by the assumptions in the Lemma, while the second inequality follows since $a_{-i} \geq a_{-i}^+$ and br_i^+ is non-decreasing. Therefore,

$$BR(\prod_i [a_i^+, \max A_i]) \subseteq \prod_i [\underline{a}_i^+, \max A_i].$$

Under condition (i) in the Lemma, Kakutani fixed point theorem implies that there exists $\bar{a} \in BR(\bar{a})$ such that $\bar{a} \geq a^+ \geq \underline{a}$. Under condition (ii) in the Lemma, $\sup BR_i(a_{-i})$ is non-decreasing in a_{-i} and Tarski fixed point theorem implies that there exists $\bar{a} = \sup BR(\bar{a}) \geq a^+ \geq \underline{a}$. In either case, $\bar{a} \in BR(\bar{a})$ and $\bar{a} \geq \underline{a}$. □

Proof of Lemma 2. Fix $\underline{a}_{-i} \leq x_{-i} \leq a_{-i}$ and for $\lambda \in [0, 1]$, consider the function

$$v(a_i, \lambda) = \lambda U_i(a_i, a_{-i}) + (1 - \lambda) u_i(a_i, x_{-i})$$

Since (2.4) holds, $v(a_i, \lambda)$ satisifes the strict single crossing condition $(a_i, \lambda) \in A_i \times \{0, 1\}$. Therefore, Theorem 4' in Milgrom and Shannon (1994) implies that

$$\inf\{BR_i(a_{-i})\} \geq \sup\{br_i(x_{-i})\} = \bar{b}r_i(x_{-i})$$

Taking supremum

$$br_i^+(a_{-i}) = \sup_{\underline{a}_{-i} \leq x_{-i} \leq a_{-i}} br_i(x_{-i}) \leq \inf BR_i(a_{-i}.)$$

□

Proof of Corollary 1. Each of the two statements in part a of the Corollary follows from parts a and b in Theorem 1.

We now prove part b. We provide a proof when u is a game with strategic complements. Since u has strategic complements, we can define $\underline{a}$ as the smallest element in NE(u). Theorem 1, part b, it follows that for any $a \in NE(U)$, $a \geq \underline{a}$. Now, let $\bar{a}$ be the largest element in NE(u). Using Theorem 1 part a, we deduce that there exists $a^* \in NE(U)$ such that $a^* \geq \bar{a}$. This completes the proof. □

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